Changes in education, wage inequality and working hours over time*

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Abstract

The US skill premium and college enrollment have increased substantially over the past few decades. In addition, while low-wage earners worked more than high-wage earners in 1970, the opposite was true in 2000. We show that a parsimonious neoclassical model featuring skill-biased technical change, endogenous education and labor supply decisions can explain the change in the US college education rate between 1967 and 2000 as well as the trend in the wage-hours correlation. Moreover, we show analytically and quantitatively that endogenous labor supply is important. Assuming constant hours significantly biases the estimates of the effects of skill-biased technological progress on college enrollment and the skill premium. Further, we find that limiting the maximum number of hours someone can work lowers welfare for almost all generations. Since it increases the skill premium, the welfare loss is most severe for the low-skilled, reaching almost one percent of life-time consumption.

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1 Introduction

Figure 1 shows three important developments that took place in the US in the last part of the 20th century. First, the skill premium, which was already 50% in the 1960s, increased by another 30 percentage points by the end of the century. Second, college enrollment increased by about ten percentage points, from 52% to 63%. Last, and probably least well-known, the male wage-hours correlation turned from negative to positive. Thus, in the 1960s poorer households worked, on average, more than richer households. Since the 1980s however, rich households have been working more than poor households. In this paper, we show why the third finding is important for better understanding the first two.

Using a standard life-cycle model with an education decision and endogenous labor supply, we show that skill-biased technological progress leads to an increase in the skill premium and the supply of skilled labor which can take two forms: on the extensive margin, more households find it worthwhile to go to college; on the intensive margin, the reward for supplying labor increases more for college-educated households. This induces them to work more than low-skilled households, leading to the increase in the wage-hours correlation. We also show that ignoring the intensive margin leads to biased estimates of the extensive margin.

While the main contribution of our paper is quantitative in nature, we also provide detailed explanations of our quantitative results by studying a simplified model analytically and by performing several counterfactual experiments. We calibrate our quantitative model to replicate the skill premium in 1967 and 2000, the enrollment rate in 1967 and the wage-hours correlation in 1967. Based on these inputs and the exogenous evolution of skill-biased technological progress, the model is able to replicate simultaneously the increase in the enrollment rate and the trend in the wage-hours correlation. Enrollment in the model rises from 52.0% in 1967 to 64.1% in 2000, very close to the data, which show an increase from 52.0% to 63.3%. The wage-hours correlation over the same period rises from −0.15 to +0.13 in the model, while it rises from −0.14 to +0.08 in the data.

We explore the mechanism behind these results qualitatively and document the importance of the working hours decisions. Three competing effects influence education decisions. First, there is the well-known *skill premium effect*. Skill-biased technological progress increases the wage premium for educated workers and thus the incentive to get an education. Second, there is a *relative hours effect*. People who have a medium learning ability but choose not to go to college in a setting where working hours are

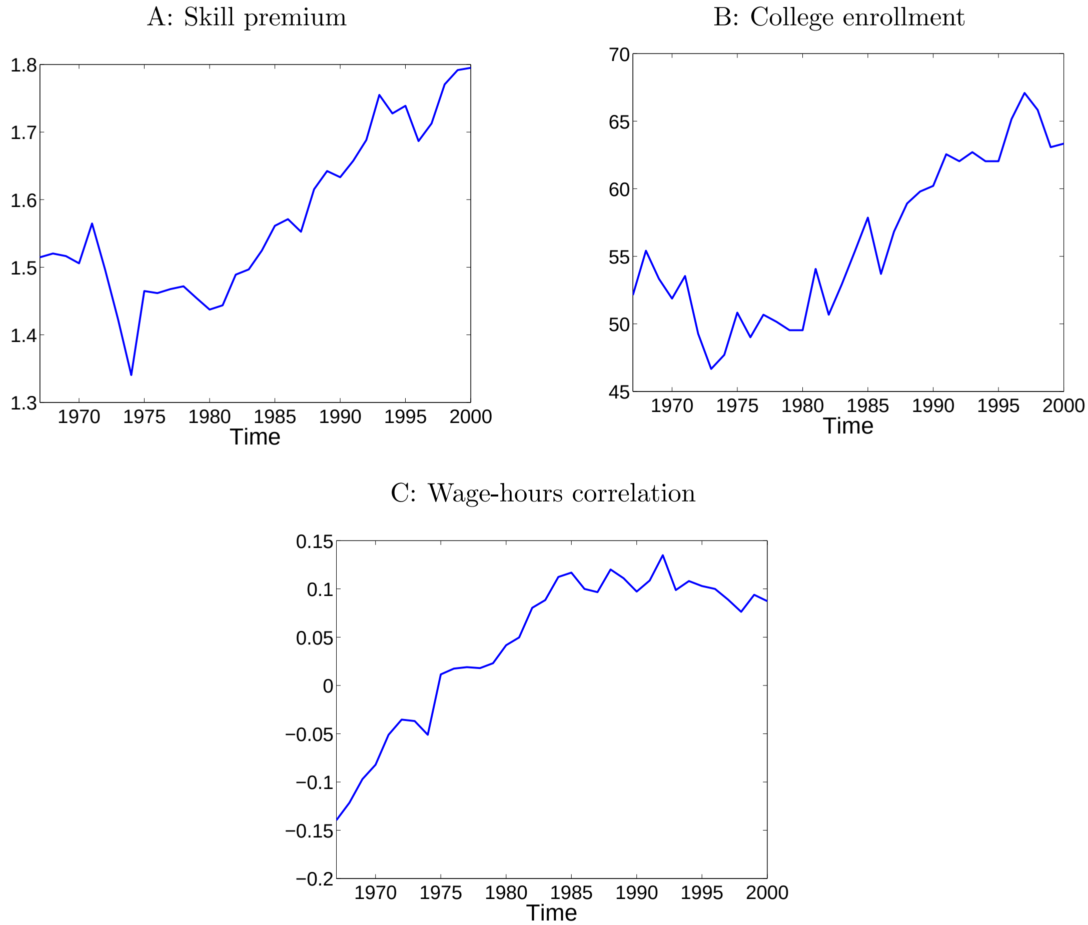

Figure 1: Three facts related to education and the labor market for males, US, 1967-2000.

Panel A shows the increase in the skill premium, defined as the wage of college graduates relative to the wage of those without a college degree. Panel B shows the variation in the college enrollment rate, defined as the percentage of high-school-educated persons enrolling in college. Panel C shows the change in the log(wage)-log(hours) correlation.

identical across skill groups may prefer to go to college when working hours can be freely chosen. The possibility of working more hours indeed increases lifetime earnings differentials, making education more attractive. Third, there is a *general equilibrium effect* operating through an extensive and intensive margin. If the freedom to choose working hours increases education, the supply of skilled workers will be larger and, in addition, college graduates might want to increase the number of hours they work. Both effects increase the supply of skilled labor, which depresses the skill premium and reduces education incentives.

In a simplified version of the model with exogenous factor price variations, we analytically document the relative hours effect. If the skill premium increases over time, high-skilled households will increase their working hours relative to low-skilled households. This effect is consistent with the increase in the wage-hours correlation, empirically exhibited in Panel C of Figure 1.

A quantitative analysis of counterfactual scenarios, where the hours decision is switched off or factor prices are exogenous, allows us to gauge the importance of the three competing effects: skill premium effect, relative hours effect and a general equilibrium effect. We find that all three effects quantitatively matter and that the third effect dominates. Ignoring hours decisions would, for instance, lead to an overestimation of the increase in college enrollment by 3 percentage points, while ignoring general equilibrium responses leads to a skill premium gap of 15 to 30 percentage points.

As a last step, we take the model to analyse working time regulations. In particular, we show that restricting the maximum time someone can work to 48 hours per week, as is done by the EU Working Time Directive, hardly changes the enrollment rate. However, the skill premium is around four percent higher in this case, implying that this regulation increases income inequality between the high-skilled and low-skilled. The main reason is that college graduates reduce their hours by more than high school graduates, which implies a lower relative supply of skilled labor than under laissez-faire conditions and thus an increased skill premium.

If the goal of the regulation is to support low-income households, this working time regulation then leads to the counterproductive outcome of higher wage inequality. A welfare analysis comes to similar conclusions: all households lose out from the regulation, except the high-skilled who are old at the time of the policy introduction. Welfare losses for young generations can, on the other hand, be sizable, at 0.7% of lifetime equivalent consumption. On the declining part of the hump-shaped life-cycle productivity profile, educated older workers work fewer hours anyway and thus are not affected by the working time constraint, and thus only benefit from the wage premium increase.

Motivated by this finding and the lower stringency of current working time regulations for high-skilled workers, we also consider a working time policy restricted to low-skilled workers and find partially reversed outcomes. Wage inequality now is slightly reduced since the high-skilled increase their labor supply while some low-skilled can not increase theirs due to the regulation. This policy also leads, on average, to lower welfare. However, the welfare losses now fall mainly on the high-skilled, and the only households which enjoy welfare gains are old low-skilled households. Thus, restrictions on hours for low-skilled lead to smaller welfare losses and to better distributional outcomes than restrictions on everyone.

Related Literature

Our paper relates to different strands in the literature on growth, education and wage inequality. To a large extent, wage inequality is driven by skill-biased technical change, which has tilted demand and supply in the labor market in favor of high-skilled households (see, for example, Katz and Murphy, 1992). Despite the increase in education levels, demand for high-skilled labor never weakened: the race (Goldin and Katz, 2009) between technological change and education has been won by the former, leading to significant increases in inequality.[1] Our paper builds on the literature that explains the increase in the skill premium through skill-biased technological progress, specifically capital-skill complementarity, as in Krusell, Ohanian, Rios-Rull, and Violante (2000), and the reduction in the price of capital equipment, as in Greenwood, Hercowitz, and Krusell (1997).

Our paper is closely related to He (2012) and Restuccia and Vandenbroucke (2013). The model by He (2012) shows that the main determinant of the increase in the skill premium is skill-biased technological progress. Demographics, from which we abstract, plays only a minor role. Restuccia and Vandenbroucke (2013), studying a longer time period, are able to replicate the increase in college education between 1940 and 1980 but overpredict the increase between 1980 and 2000 by more than 40 percent. In both of these models, labor supply is exogenous. Thus, they cannot investigate either wage-hours correlation variations or the influence of freely chosen hours on education.

Three quantitative studies that include endogenous labor supply are Heathcote, Storesletten, and Violante (2010, 2014) and Guvenen and Rendall (2015). Guvenen and Rendall (2015) estimate a structural partial equilibrium model to explain education, marriage, divorce, and labor market decisions jointly. While their main focus is somewhat different, their model can speak to the link between hours and education. However, their model predicts a counterfactually large drop in male hours and underes-

[1] See the reviews by Lemieux (2008) or Acemoglu and Autor (2011) for further details.

timates their college enrollment. Our model shows that these two results are intimately related. Heathcote, Storesletten, and Violante (2010) use a model with endogenous education and hours decisions to investigate the impact of wage inequality on gender, hours and consumption inequality, successfully accounting for the variation (but not the level) in the wage-hours correlation between 1967 and 2005. Heathcote, Storesletten, and Violante (2014), on the other hand, succeed in matching both variations and levels of the wage-hours correlation between 1968 and 2006 but need fluctuations in cohort-specific heterogeneous preferences for labor to obtain this result.

Marimon and Zilibotti (2000) study the effect of working time regulation on employment and unemployment levels in a matching framework, which is something we abstract from. They find that small reductions in hours worked can lead to small increases in employment, while larger reductions lead to a fall in employment due to the diminished incentives to post vacancies. Their analysis, however, does not consider the impact on education or its influence on labor market outcomes. Yurdagul (2017) studies the effects of working time regulation in a model with production complementarities and where households have to to decide between becoming workers or entrepreneurs. He finds that hours restriction do not lead to a fall in the number of workers but to a fall in the number of entrepreneurs. The model however abstracts from education decisions.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 provides analytical results for a simplified version of the model. Section 4 describes the calibration and the quantitative results and explains the importance of endogenous labor supply. Section 5 investigates the impact of workweek regulations. Section 6 concludes.

2 Model

We build a neoclassical, deterministic overlapping-generations model with skill-biased technical change as well as endogenous hours and education decisions in general equilibrium. Following Greenwood, Hercowitz, and Krusell (1997) and Krusell, Ohanian, Rios-Rull, and Violante (2000), we model skill-biased technical change as the result of capital-skill complementarity and variations in the price of capital equipment.[2] As in Heathcote, Storesletten, and Violante (2010), education decisions depend on wage

[2]The literature sometimes calls this investment-specific technological change (ISTC).

differentials and ex-ante heterogenous learning ability.[3]

2.1 Households

Population We abstract from population growth so that the size of the economy is constant. Households become economically active after high school when they start their adult life, at which point we say that they are *born*. At that point in time a household decides whether or not to go to college. If he does not attend college, he starts to work as an unskilled worker immediately. If he enrolls in college, he will graduate four years later and will then start to work as a skilled worker. All households work until they die, J periods after birth, leaving neither debt nor bequests. As there is no bequest motive, all households are born with no assets.

Preferences Lifetime utility from consumption and leisure of a household born in time v is given by

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{v,v+j-1}, l_{v,v+j-1}) \tag{1}$$

where β is the discount factor, $c_{v,t}$ denotes consumption of the household at time t and $l_{v,t}$ its labor supply. We assume that the instantaneous utility function is separable and given by

$$u(c,l) \equiv \ln(c) - \gamma \frac{l^{1+\epsilon}}{1+\epsilon}, \tag{2}$$

where ϵ is the inverse of the (Frish) elasticity of labor supply. These preferences have the well-known implication that income and substitution effects cancel out for permanent wage changes. However, we analyze the transition path of a sequence of skill-biased technology shocks which lead to a new steady-state. The relative strength of income and substitution effects changes during this transition, leading to changes in hours worked.

Education Households are born with different learning abilities. They draw an ability $i \in [0;1]$ from a uniform distribution at birth. Education comes at a utility cost $\chi(i)$, capturing psychological costs of learning in reduced form and at the cost of foregone wages during college. The education disutility cost function declines with ability, $\chi' < 0$: households born with high ability i find education easy and are more likely to choose college education. We denote college education by $s_i = c$ and high school education by $s_i = h$.

[3] The resulting model is similar to He (2012) except that our model does not feature demographics but includes endogenous labor supply.

Productivity and budget constraints Wage differences arise not only from skill differences and changes in the skill premium but also from age efficiency profiles. We assume separate exogenous time-invariant profiles $\left\{\varepsilon_j^s\right\}$ for each education level s. This implies the following budget constraint

$$a_{v,v+j} + c_{v,v+j-1} = \left(1 + r_{v+j-1}\right) a_{v,v+j-1} + \left(1 - \mathcal{I}_{v,v+j-1}\right) w^s_{v+j-1} \varepsilon^s_j l_{v,v+j-1}, \tag{3}$$

where

$$\begin{aligned} \mathcal{I}_{v,v+j-1} &= 1 \qquad \text{if} \quad s = c \ \wedge \ j \leq 4 \\ \mathcal{I}_{v,v+j-1} &= 0 \qquad \text{otherwise.} \end{aligned}$$

Thus, the indicator variable captures the fact that households that go to college forgo labor income while studying. Factor prices are time-varying but, as aggregate variables, have no age subscript. For simplicity we ignore the pecuniary costs of education.[4] Despite this, households who go to college have to borrow to finance early-life consumption since they have no income.[5]

Households maximization problem Conditional on the education choice s, the maximization problem of a household born at time v, of age j and alive in period $v + j - 1$ can be expressed as

$$V^s_{v,v+j-1}\left(a_{v,v+j-1}\right) = \max_{c_{v,v+j-1}, l_{v,v+j-1}} \ u\left(c_{v,v+j-1}, l_{v,v+j-1}\right) \ + \ \beta V^s_{v,v+j}\left(a_{v,v+j}\right) \tag{4}$$

and is subject to the budget constraint (3). In the first period of their life (at $j = 1$), households decide whether to go to college or whether to remain unskilled by comparing the corresponding value functions. The resulting choice is given by

$$s = \begin{cases} c & \text{if} \quad V^c_{1,t}\left(0\right) - \chi(i) \geq V^h_{1,t}\left(0\right) \\ h & \text{if} \quad V^c_{1,t}\left(0\right) - \chi(i) < V^h_{1,t}\left(0\right). \end{cases} \tag{5}$$

[4]We also considered cases with tuition costs. Since these models had similar implications, we do not report them here. See He (2011, 2012) for models that incorporate tuition costs.

[5]Since households in the model do not face risks, they can borrow at the risk-free rate up to the natural borrowing limit, which in our model never becomes binding.

2.2 Markets

Labor market and production A representative firm uses capital K, skilled labor S and unskilled labor U to produce a single good, paying factors their marginal product. Skilled and unskilled labor are imperfect substitutes because of capital-skill complementarity. The aggregate production function is

$$Y_t = F(K_t, S_t, U_t) = \left[\mu U_t^{\theta} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda) S_t^{\rho}\right)^{\theta/\rho}\right]^{1/\theta}, \tag{6}$$

with $\mu, \lambda \in (0,1)$ and $\rho < \theta < 1$. The condition $\rho < \theta$ implies capital-skill complementarity, the elasticity of substitution between skilled labor and capital $1/(1-\rho)$ being smaller than the elasticity of substitution between unskilled labor and the capital-skilled labor aggregate $1/(1-\theta)$.

Production capital is derived from households' assets

$$A_t = \sum_{s \in \{c,h\}} (A_t^s) \equiv \sum_{s \in \{c,h\}} \left(\sum_{v=0}^{t} N_{v,t}^s a_{v,t}^s \right) \tag{7}$$

where $N_{v,t}^s$ is the number of households who choose education s, born at time ν and alive at time t and $a_{\nu,t}^s$ are their assets. Firm capital depreciates at rate δ. Savings (investment) $X_t = A_{t+1} - A_t$ can be transformed into production capital K_t thanks to a technology which is exogenously improving over time and represented by a price q_t. The law of motion for capital is

$$K_{t+1} = (1-\delta) K_t + X_t q_t. \tag{8}$$

The exogenous time-invariant labor input efficiency process $\left\{\varepsilon_j^s\right\}$ depends on the skill level and on age. The aggregate efficient labor supply of the unskilled and skilled workers are given, respectively, by

$$U_t = \sum_{v=0}^{t} N_{v,t}^h \varepsilon_{t-\nu}^h l_{v,t}^h \qquad S_t = \sum_{v=0}^{t} N_{v,t}^c \varepsilon_{t-\nu}^c l_{v,t}^c. \tag{9}$$

Goods market Defining aggregate consumption C_t in a way similar to aggregate assets A_t, the goods market must clear:

$$Y_t = C_t + X_t. \tag{10}$$

We provide first order conditions with other equilibrium properties and a formal definition of the competitive equilibrium. We then continue with an analysis of the role of hours in a simplified version of the model, which exhibits an intertemporal relative labor supply effect, at the heart of our study.

2.3 Equilibrium properties

Change of variable We perform a permanent change of variable, borrowed from Greenwood, Hercowitz, and Krusell (1997), which simplifies the analysis: $\tilde{K}_{t+1} \equiv K_{t+1}/q_t$. Then the production function (6) and the capital law of motion (8) are equivalent to

$$Y_t = \left[\mu U_t^\theta + (1-\mu)\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\,S_t^\rho\right)^{\theta/\rho}\right]^{1/\theta},$$

$$\tilde{K}_{t+1} = \left(1-\tilde{\delta}\right)\tilde{K}_t + X_t,$$

with additional notation $B_t = q_{t-1}$ and $\tilde{\delta} = 1-(1-\delta)q_{t-1}/q_t$. After the change of variable, the capital law of motion has the familiar neoclassical growth expression. Assets market clearing $\tilde{K}_t = A_t$ then implies goods market clearing (10), a property which simplifies the numerical implementation of the model.

Optimality conditions First order conditions, applying to all life-cycle periods of the household, are given by:

$$\frac{c^s_{v,t+1}}{c^s_{v,t}} = (1+r_{t+1})\beta \qquad\qquad l^s_{v,t} = (1-\mathcal{I}_{\nu,t})\left(\frac{1}{\gamma}\frac{w^s_t\varepsilon^s_{t-\nu}}{c^s_{v,t}}\right)^{\frac{1}{\epsilon}}.$$

The first condition is the Euler equation. The only role of the term $(1-\mathcal{I}_{\nu,t})$ in the second condition is convenience in notation: when households obtain education ($\mathcal{I}_{\nu,t} = 1$), they do not work.

Factor prices As factors are paid their marginal products, the net interest rate and wage rates are equal to:

$$\begin{aligned}
r_t &= \lambda\,(1-\mu)\,B_t^\rho H_t\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\,S_t^\rho\right)^{\theta/\rho-1}\tilde{K}_t^{\rho-1} - \tilde{\delta}\\
w_t^c &= (1-\mu)\,(1-\lambda)\,H_t\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\,S_t^\rho\right)^{\theta/\rho-1} S_t^{\rho-1} \\
w_t^h &= \mu H_t U_t^{\theta-1}
\end{aligned} \tag{11}$$

where $H_t = \left[\mu U_t^\theta + (1-\mu)\left(\lambda\left(B_t\tilde{K}_t\right)^\rho + (1-\lambda)\,S_t^\rho\right)^{\theta/\rho}\right]^{1/\theta-1}$.

Skill premium From (11), the skill premium follows:

$$\frac{w_t^c}{w_t^h} = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\lambda\left(\frac{B_t\tilde{K}_t}{S_t}\right)^{\rho} + (1-\lambda)\right)^{\theta/\rho-1}\left(\frac{S_t}{U_t}\right)^{\theta-1}.$$

The expression illustrates the impact of skill-biased technical change, or skilled labor demand, and skilled labor supply on the skill premium. Since $\mu, \lambda \in (0,1)$ and $\rho < \theta < 1$, a larger supply of skilled labor S_t depresses the skill premium, ceteris paribus. Increases in the demand for skilled labor follow from increases in $B_t = q_{t-1}$, which represents an improvement in investment technology and leads to skill-biased technical change. When it happens the skill premium increases, ceteris paribus. These impacts on the skill premium are familiar and expected.

Competitive equilibrium A competitive equilibrium over the time periods $\mathcal{T} = \{t_0, t_0+1, ..., t_0+T\}$ is a sequence

$$\left\{\ s_{\nu,t}, c_{v,t}, l_{v,t}, w_t^c, w_t^h, r_t, N_{v,t}^s \ \mid v, t \in \mathcal{T}\ \right\}$$

such that education decisions $s_{\nu,t}$ solve (5), consumption and leisure decisions $(c_{v,t}, l_{v,t})$ solve the Bellman equations (4) subject to budget constraints (3), factor prices (w_t^c, w_t^h, r_t) are paid their marginal products as summarized in conditions (11), factor markets clears as expressed in (7) and (9) and the good market clears as expressed in (10).

3 Wage inequality and hours: analytical results

We use a simplified version of the model to analyse the impact of wage inequality on the dispersion of working hours, exhibiting a relative intertemporal labor supply effect.

We consider a partial equilibrium version of the model with a constant interest rate and an exogenous skill premium variation given by

$$w_t^c = (1+\alpha_t)\, w_t^h$$

for a given path of the skill premium $\alpha_t \geq 0$. We also assume that education does not take time and an identical age efficiency profile for unskilled and skilled workers ($\varepsilon_{t-\nu}^h = \varepsilon_{t-\nu}^c$ for all $t \geq \nu$).

First we express optimality conditions for the simplified model and derive its implications. Consider two households born at the same time but making different education choices. The first order conditions for labor can be rewritten as $c_v^s(t) = \frac{1}{\gamma}\frac{1}{(l_t^s)^\epsilon} w_t^s \varepsilon_{t-\nu}^s$. Comparing the consumption decisions of the two households,

$$\frac{c_v^c(t)}{c_v^h(t)} = \left(\frac{1}{\gamma}\frac{w_t^c \varepsilon_{t-\nu}^c}{(l_v^c(t))^\epsilon}\right) \Big/ \left(\frac{1}{\gamma}\frac{w_t^h \varepsilon_{t-\nu}^h}{(l_v^h(t))^\epsilon}\right) = \left(\frac{l_v^h(t)}{l_v^c(t)}\right)^\epsilon \frac{w_t^c}{w_t^h} = \left(\frac{l_v^h(t)}{l_v^c(t)}\right)^\epsilon (1+\alpha_t).$$

From the Euler equation it follows that consumption grows at the same speed for the two households. Hence the quantity expressed above is constant. Lemma 1 consolidates optimality conditions and sums up the additional derivations:

Lemma 1 ***(Optimality conditions):*** *For households born at time v, optimal labor supply and consumption decisions are characterized by the following:*

$$l_v^s(t) = \left(\frac{w_t^s \varepsilon_{t-\nu}^s}{\gamma c_v^s(t)}\right)^{\frac{1}{\epsilon}} \qquad \frac{c_v^s(t+1)}{c_v^s(t)} = (1+r_{t+1})\,\beta \qquad \left(\frac{l_v^h(t)}{l_v^c(t)}\right)^\epsilon (1+\alpha_t) = \frac{c_v^c(t)}{c_v^h(t)} = constant.$$

Assume that the skill premium α increases over time. The last relationship says that labor supply l^h of the low-educated will increase more slowly than labor supply l^c of the high-educated, a *relative intertemporal labor supply effect*. Intuitively, it makes more sense for high-educated households to work more than low-educated households in the future rather than in the present, when they earn an even higher wage. If the premium increases for a long enough period, the high-educated will eventually work more, consistent with the wage-hours correlation shown in Figure 1.

We provide formal derivations of this intuitive discussion in Lemmas 2 and 3. The first shows that labor supply is the same for households when the skill premium is constant over their lifetime. It also shows that high-educated households work less at the start of their life, and more at the end, if the skill premium never decreases and increases at least once over their lifetime.

Lemma 2 ***(Relative hours):*** *Compare an educated household with an uneducated household born at the same time v. Then...*
(a) if the skill premium is constant over their lifetime ($\dot{\alpha} = 0$), households work the same (and consume according to their productivity):

$$l_v^c(t) = l_v^h(t) \qquad c_v^c(t) = (1+\alpha_v)\,c_t^h(t) \qquad t \in [v, v+J]$$

(b) if the skill premium never decreases and increases once or more over their lifetime ($\dot{\alpha} \geq 0$ with $\dot{\alpha} > 0$ at least once), educated households initially work less than uneducated

households, but more before they die: there exists t_1 and t_2 such that

$$l_v^c(t) < l_v^h(t) \quad \forall v \leq t < t_1 \qquad\qquad l_v^c(t) > l_v^h(t) \quad \forall t_1 < t_2 \leq t \leq v + J.$$

Further, $l_v^c(t)/l_v^h(t)$ is constant over intervals where the skill premium is constant.

Proof : See Appendix A.1.

The converse to case (b) of the lemma also holds: if the skill premium never increases and decreases at least once of the lifetime of the two households, educated households initially work more than uneducated households, and then less. The intuition for case (a) of the lemma is the following: in absence of transitory shocks to wages, substitution and income effects cancel out so labor supply is identical, and lifetime income and consumption differences between educated and uneducated households equal the productivity difference (skill premium). The intuition for case (b) is based on the relative intertemporal labor supply effect and case (a): when the premium is constant, households work an equal number of hours; when the premium increases, educated households work more when the premium is higher, which is towards the end of their life.

Lemma 3 makes a formal connection between relative labor supply and the wage-hours correlation.

Lemma 3 ***(Wage-hours correlation):*** *Consider an economy where the skill premium is constant except it increases once ($\alpha_t = \alpha_0$ for $t < t_1$ and $\alpha_t = \alpha_1 > \alpha_0$ for $t \geq t_1$). Then the wage-hours correlation coefficient is first negative and then positive (as long as generations which experienced the skill premium increase are alive):*

$$corr\left(w_t^i, l_t^i\right) < 0, \quad t_1 - J \leq t < t_1; \qquad\qquad corr\left(w_t^i, l_t^i\right) > 0, \quad t_1 \leq t \leq t_1 + J.$$

Proof : See Appendix A.1.

The intuition for the result comes from Lemma 2: when the skill premium is lower, educated households, who have a higher wage, work initially less than uneducated households, making the wage-hours correlation negative; before they retire, they work more, changing the sign of the correlation. The converse of the result also holds: if the skill premium is constant except it for a one-time decrease, the wage-hours correlation is first positive and then negative.

The proof can neither be easily extended to the correlation between hours and efficient wages, which are observed in practice, nor can it be extended to the case where the skill premium increases more than once since multiple generations are involved. Yet in reality the skill premium has varied continuously. Therefore, we analyse the effect of multiple changes in the skill premium in a quantitative model in Section 4. However, Lemma 3 still formally articulates a mechanism where increases in the skill premium contribute to the increase (and change of sign) in (efficient) wage-hours correlations, as witnessed in the US towards the end of the 20th century and documented by Costa (2000) and Heathcote, Perri, and Violante (2010).

4 Quantitative results

In the previous section, we have shown analytically how an increase in the skill premium can lead to (i) an increase in labor supply of high-skilled households and (ii) an increase in the wage-hours correlation. In this section, we analyse whether these outcomes also matter quantitatively and whether the observed increase in the skill premium since the Second World War can explain (i) the increase in college enrollment rates and (ii) the increase in the wage-hours correlation. Our calibration strategy is to use data for the beginning of our sample period (1967), estimates of the price of capital between 1967 and 2000, and wage data for 2000 to match the change in the skill premium perfectly. The resulting changes in college enrollment rates and wage-hours correlation are equilibrium outcomes and used to evaluate the model's performance. We first discuss our calibration in Section 4.1. Then, we show our baseline results in Section 4.2 before discussing the role of endogenous labor supply in Section 4.3.

4.1 Calibration

Our calibration strategy has two components. One group of parameters, shown in Panel A of Table 1, is based on previous research. Another group of parameters, shown in Panel B of Table 1, is calibrated to match crucial stylized facts.[6]

Since households enter in the model only when joining the labor market or starting education (age 18) and exit after they stop working (age 65), we follow He (2012) and set lifetime duration J to 48. Age efficiency profiles for educated $\left\{\varepsilon_j^c\right\}$ and uneducated workers $\left\{\varepsilon_j^h\right\}$ are derived from average wages in the second half of the century, as reported in Figure 8 in He (2012). One period corresponds to one year, so we set the

[6]Further details on the calibration and the solution algorithm can be found in Appendix A.2.

Table 1: Exogenous model parameters

	Parameter	Value	Source
	A: Exogenous parameters		
J	Lifetime duration	48	Real average working life
$\left\{\varepsilon_j^s\right\}$	Age efficiency profiles	...	He (2012) based on CPS
β	Time discounting factor	0.96	Aiyagari (1994)
$1/\epsilon$	Labor supply elasticity	1	Shimer (2009)
$\{q_t\}$	Price of capital equipment 1967-2000	...	Cummins and Violante (2002)
δ	Capital depreciation rate	0.069	Imrohoroglu et al. (1999)
θ	Elasticity param. unskilled labor/capital	0.401	Krusell et al. (2000)
ρ	Elasticity param. skilled labor/capital	-0.495	Krusell et al. (2000)
	B: Calibrated parameters		
η	Mean in dist. education disutility	0.161	
κ	Variance in dist. education disutility	2.5	
γ	Scale labor supply disutility	7.52	Calibrated jointly
μ	Share for unskilled labor in production	0.396	
λ	Share for capital in production	0.870	

time discount factor β to 0.96.[7] There is a debate in the literature for the value of labor supply elasticities: we follow Shimer (2009) and choose a middle value of 1. We thus set ϵ such that the elasticity $1/\epsilon = 1$.

The price of capital equipment $\{q_t\}$ between 1967 and 2000 is derived from the estimates of the quality-adjusted prices for total investment from Cummins and Violante (2002). The capital depreciation rate δ is taken from Imrohoroglu, Imrohoroglu, and Joines (1999). The parameters θ and ρ which defines the elasticity of substitution between capital, skilled and unskilled labor are taken from Krusell, Ohanian, Rios-Rull, and Violante (2000).

Following Heathcote, Storesletten, and Violante (2010), we assume that the utility cost function for education follows a log-normal distribution with mean η and variance κ. Therefore, we have five parameters in Panel B of Table 1 that need to be determined jointly. These are the scale factor for labor supply disutility γ, the share factors for unskilled labor and the capital-skilled labor composite in the production function μ and λ in addition to η and κ.

As mentioned before, we calibrate these five parameters to match four moments in the initial steady state in 1967 and one moment in 2000. Table 2 shows the target

[7] The resulting capital-output ratio K/Y is 3.14 in the initial steady-state.

Table 2: Model moments

Variable	Model	Data
log wage - log hours correlation in 1967	-0.15	-0.14
Average number of hours worked in 1967	0.37	0.37
College enrollment rate in 1967	0.52	0.52
Skill premium in 1967	1.52	1.52
Skill premium in 2000	1.80	1.84

variables, their data counterparts and the model outcomes. Overall, the model fits the data well. The initial wage-hours correlation is negative, implying that the high-skilled worked less hours than the low-skilled. The college enrollment rate is 52% in the model and in the data.[8] The skill premium in the model is 52% (80%) in 1967 (1980). The early number is matched perfectly, while the latter is slightly lower than in the data. The quality of the calibration will be assessed in the next section when we look at two moments that were not targeted.

4.2 Simulation results

Given variations in the price of capital equipment, our model makes predictions on wages, education and hours decisions. Using estimates of the price of capital equipment between 1967 and 2000, we generate time series for the skill premium, college enrollment decisions and the wage-hours correlation in the United States and compare them with empirical observations. The results are shown in Figure 2, where the model predictions are shown with dashed lines and the data with solid lines.

The calibration strategy implies that the model outcomes for enrollment and wage-hours correlation are equal to the data in 1967 and for the skill premium in 1967 and 2000. Enrollment and the wage-hours correlation in 2000, however, are model outcomes, as is the time-path between 1967 and 2000 of all variables, including the skill premium.

Panel A in Figure 2 shows indeed that the time-path of the skill premium is close to the empirical trend, except for the initial 5 years drop which the model fails to reproduce. Behind this trend for the skill premium is the continuous fall in the relative price of capital equipment which benefits high-skilled workers. Since the relative price does not show a drop in the first few years, the model fails to reproduce the initial drop

[8] Following Heathcote, Perri, and Violante (2010) and He (2012), we classify workers with a college degree as skilled and workers with a high school degree and no college as unskilled. This leaves those who attended college but did not obtain a degree. We classify half of the workers with some college but no college degree as skilled and the other half as unskilled. Our resulting college enrollment data is identical to Figure 4 in He (2012).

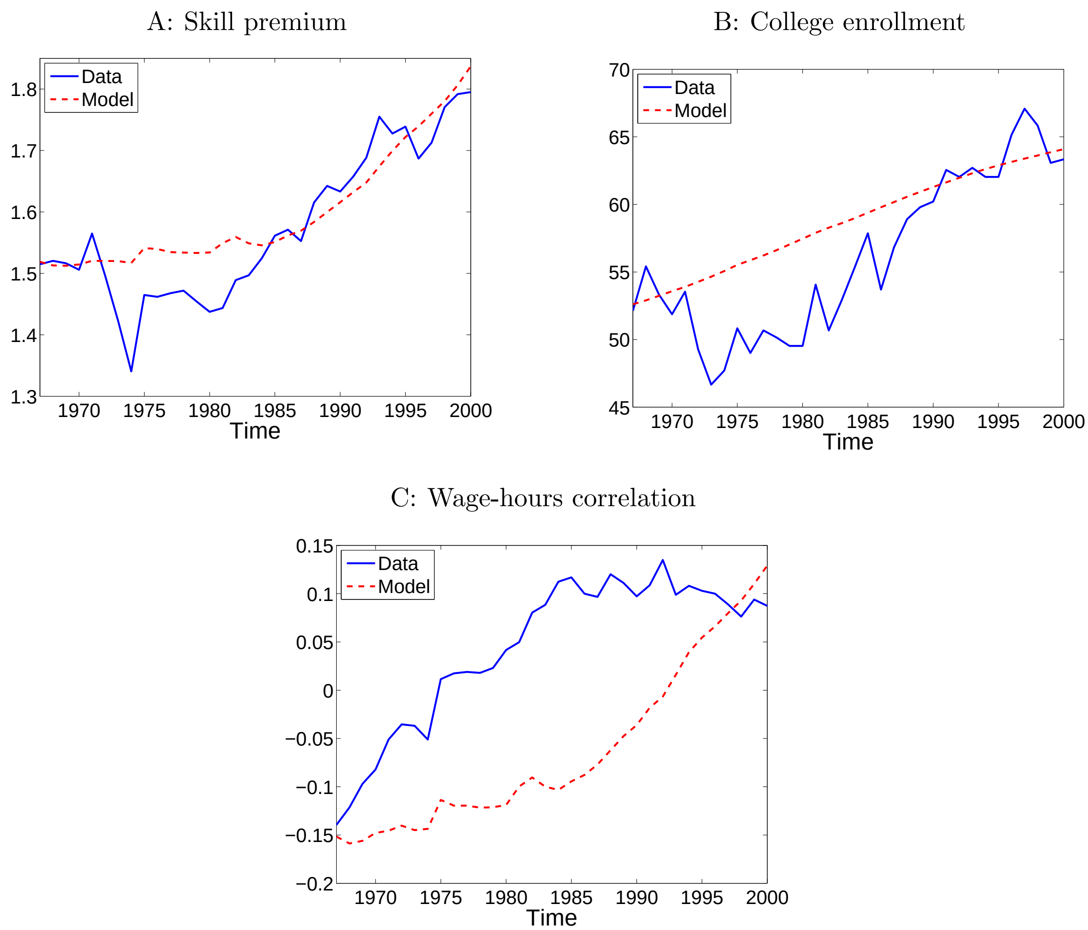

Figure 2: Data and model predictions for education and the labor market.
Panel A shows the increase in the skill premium, defined as the wage of college graduates relative to the wage of those without a college degree. Panel B shows the variation in the college enrollment rate. Panel C shows the change in the log(wage)-log(hours) correlation.

observed in the data.[9]

Panel B shows not only that the model generates an increase in college enrollment endogenously but also that the magnitude is very close to the data. The enrollment rate in 2000 in the model is 64.1% while it is 63.3% in the data. Thus, even though this moment was not targeted, the model generates a very similar result. Similar to the skill premium, the time-path in the model is smoother and, in particular, misses the initial drop. The reason for this is again that the evolution of the relative price of capital equipment is fairly smooth, generating smooth adjustments in the premium and enrollment.

Panel C shows that the model is also able to reproduce the trend in the wage-hours correlation. While it was -0.15 in 1967, it is 0.08 in the data and 0.13 in the model in 2000. This switch in sign is the second important moment that the model replicates successfully. Note, however that the time-path in the model is somewhat off. In the model, it takes more time until high-skilled households increase their hours sufficiently for the wage-hours correlation to become positive. The main reason follows from the initial gap between model and data on education, discussed previously with respect to Panel B. Because the model initially overestimates education decisions, the skilled labor force is on average younger in the model than in the data. Given the hump-shaped productivity profile, young workers provide less labor. These two features explain the slow initial increase in hours by high-skilled households and the resulting slow increase in the wage-hours correlation.

4.3 The role of the choice of hours

In this subsection we look at the role of the choice of hours for the enrollment decision and the wage-hours correlation. There are three competing effects on the enrollment decision: a *skilled premium effect*, a *relative hours* effect and a *general equilibrium* effect. The first two operate at the individual level and the last one is a feedback effect.

First, an increase in the skill premium increases the return on costly education investments and thus strengthens education incentives. Second, suppose hours and the skill premium were exogenously fixed. If the marginal person whose disutility of education is slightly too large to go to college were able to work longer hours, he would be able to generate a larger income and therefore might decide to go to college. Thus, for a given skill premium, the possibility to work more hours increases the incentive to obtain an education. Third, the skill premium will, however, not remain constant

[9] He (2012) shows that demographics can explain this drop to a significant degree.

in general equilibrium. The supply of skilled labor will increase at the intensive and extensive margin. Those with a low disutility of education who would have enrolled in college anyway now choose to work more hours when the skill premium increases, which, in general equilibrium, depresses the skill premium.

The interplay of these three effects is also evident in the increase in the wage-hours correlation and shows the importance of studying the skill premium, wage-hours correlation and enrollment jointly.

We evaluate the contribution of the three effects with the following experiments, the results of which are shown in Figure 3. The experiments are carried out in three steps. The first step will illustrate the skill premium effect. The first and second steps will jointly illustrate the importance of endogenous labor supply and the relative hours effect. The second and third steps will jointly illustrate the influence of the third channel: general equilibrium effects.

As a first step, we calibrate a model version with constant hours.[10] The results are shown in Panels A(i)-(iii). Since we still target the skill premium in 2000, the evolution of the skill premium in Panel A(i) is very similar to the benchmark case. As before the enrollment rate is endogenous. Panel A(ii) shows that it increases to 67%, three percentage points higher than in the data and the benchmark model. Panels A(i) and A(ii) illustrate the skill premium effect, the increase in education incentives as the wage differential between high-skilled and low-skilled workers becomes larger over time. Since hours are constant, everyone works the same number of hours and the wage-hours correlation in Panel A(iii) is zero.

The second step is shown in Panels B(i)-(iii), where we perform the following partial equilibrium experiment: We keep all parameters and the evolution of factor prices as in the model with constant hours but allow households to decide on their hours and education. The skill premium in B(i) is thus identical to the one in A(i). Panel B(ii) shows that in this scenario, the enrollment rate is significantly higher than before in all periods. For example, the enrollment rate with endogenous hours is three percentage points higher than under exogenous hours in 2000. The response at the extensive margin is strong. The experiment shows that the relative intertemporal labor supply effect, exhibited analytically in Section 3, matters quantitatively: ceteris paribus (under partial equilibrium with an exogenous skill premium), education incentives are increased if educated households can boost the return on education by working more hours. This also implies that ignoring endogenous labor supply responses can lead to a bias in college enrollment of up to three percentage points. Moreover, there is a significant

[10]Details can be found in Appendix A.3.

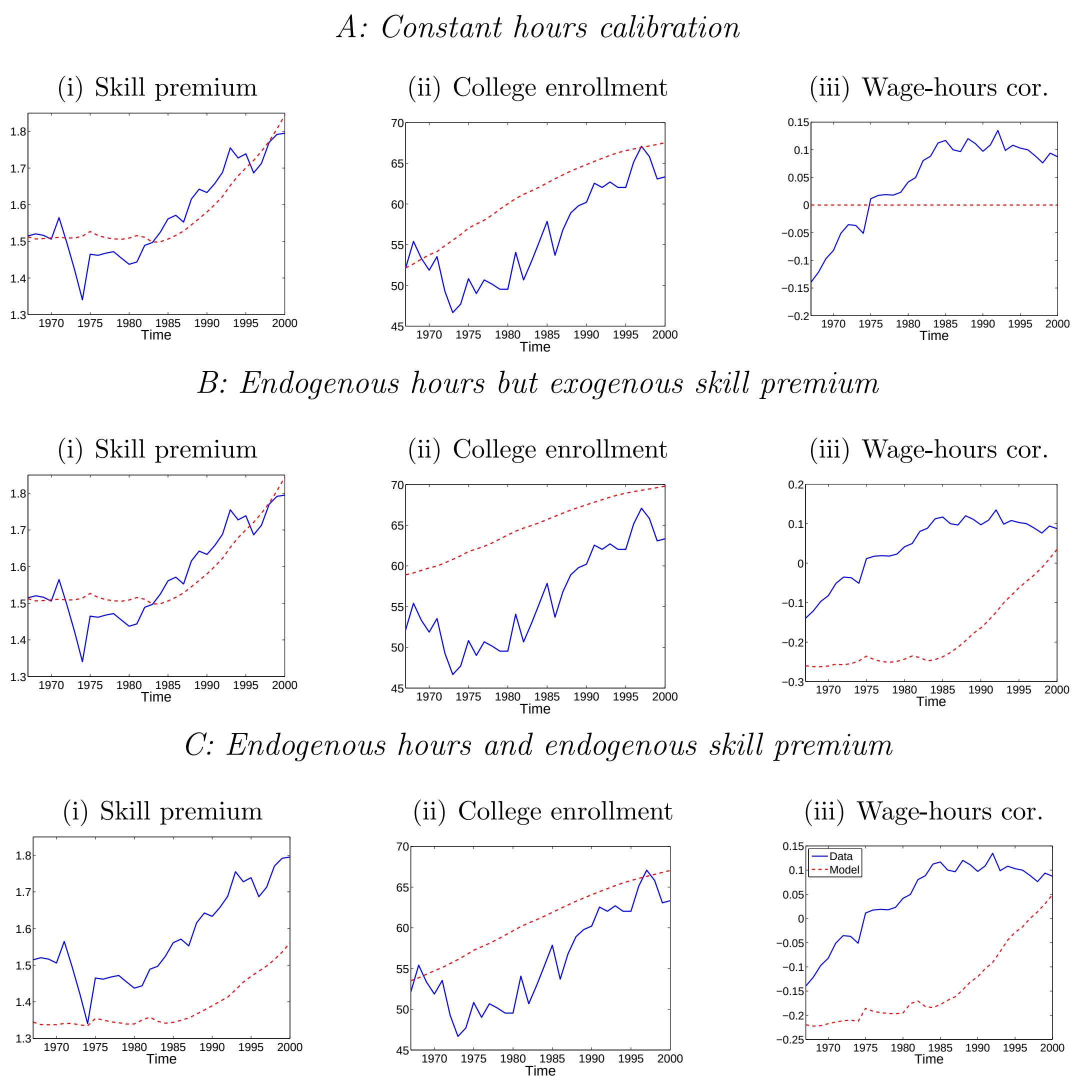

Figure 3: Inspecting the mechanism.
Row A shows the main results for the constant hours calibration. Row B shows the partial equilibrium outcome where factor prices remain at the values from Row A but agents can choose hours. Row C shows the general equilibrium outcome when hours and factor prices can adjust.

response at the intensive margin. Those households who obtain an education work on average 12% more than those who remain unskilled (unreported in Figure 3). The effect at the intensive margin is behind the increase in the wage-hours correlation shown in B(iii). Initially, the wage hours correlation is (too) negative. But over time, as the rise in the skill premium induces high-skilled to increase their labor supply, the correlation becomes slightly positive.

In the third step, we continue the previous, partial equilibrium experiment but now impose market clearing. In partial equilibrium, the increase in the skill premium led to positive effects on skilled labor supply at the intensive and at the extensive margin. One can thus clearly expect that the skill premium will fall in the general equilibrium setting. Since a smaller increase in the skill premium implies lower incentives to go to college, the supply of skilled agents will fall, as will their hours worked, which will dampen the general equilibrium effect. The results are shown in Panels C(i)-(iii).[11]

Panel C(i) shows that the general equilibrium effect on the skill premium is large. The premium falls in all periods between 15 and 30 percentage points. It is for example 55% instead of 85% in 2000. The drop in the skill premium takes place despite the fact that the smaller skill premium leads to an endogenous fall in the enrollment rate of about three percentage points in all periods, which can be seen in Panel C(ii). The effect on the intensive margin is also important for skilled agents. The average hours worked of the high-skilled increase only by about 7% instead of 12% (unreported). The correlation between wages and hours in Panel C(iii) nevertheless shows a similar pattern.

Overall, these results demonstrate the importance of analyzing the skill premium, the extensive margin (enrollment) and the intensive margin (hours) of skilled labor supply jointly. The joint analysis is important not only to understand the evolution of the wage-hours correlation but also to understand the quantitative effects of skill-biased technological progress. By assuming constant hours, one ignores the incentive for the high-skilled to increase their labor supply which, through a general equilibrium effect, also counteracts the effects of skill-biased progress on the skill premium. This implies that, for a given change in technology, the effect on wage inequality is overstated. As Panel C(i) shows, this bias can be as high as 30 percentage points.

[11]Models in Figure 2 and Panel C of Figure 3 are identical but differ in their calibration. The baseline results appear in Figure 2 because the calibration approach in Panel C of Figure 3 serves decomposition purposes only.

5 Regulating hours

In this section, we analyse the effect of regulating the maximum number of hours someone can work over a given time period. Working time regulation has been existing since the 19th century at least and has generally capped the length of the work week. However, even today, there are significant differences across countries. Since 1993, the EU Working Time Directive has, however, introduced a degree of harmonization between countries of the European Union (for details, see Marimon and Zilibotti, 2000). One element of this regulation is to limit the maximum hours a worker can work to 48 hours per week. The justification for this type of regulation is usually health and safety issues, about which our model has nothing to say. However, since our model features imperfect substitutability between different educational groups, it allows us to analyze the macroeconomic side effects of this regulation on education and inequality. In Section 5.1, we analyze the effects of imposing a maximum number of hours on every worker in the economy. We will see that this regulation hurts most generations in our model economy, with particularly strong negative effects on the low-skilled. Therefore in Section 5.2, we analyze the effects of imposing a maximum number of hours on one skill group only.

5.1 Uniform limit on hours worked

The only addition we need to make to the model is the following restriction to equation (3)

$$l \leq l^{max}, \tag{12}$$

where we set $l^{max} = 0.41$ to reflect the EU regulation.[12] All other parameters are left at their baseline values. In Figure 4, we show the path of key variables had the US adopted this regulation from 1967 onwards.[13]

Overall, the results of the simulations are similar to the baseline model. The enrollment rate is hardly affected, while the college premium with regulated hours is, on average, 1.5 percentage points higher. The reason for limited effects is that both groups, the college educated and high school graduates, are affected similarly: in the baseline, the maximum hours of college-educated workers is around 0.465 and the maximum for

[12] The 0.41 value corresponds to the average number of hours worked in 1967 in the US in model (0.37) multiplied by the EU regulation maximum (48.0) divided by the average workweek duration (43.4) in 1967.

[13] Introducing the regulation at a later date would affect the quantitative results somewhat, but the pattern remains the same.

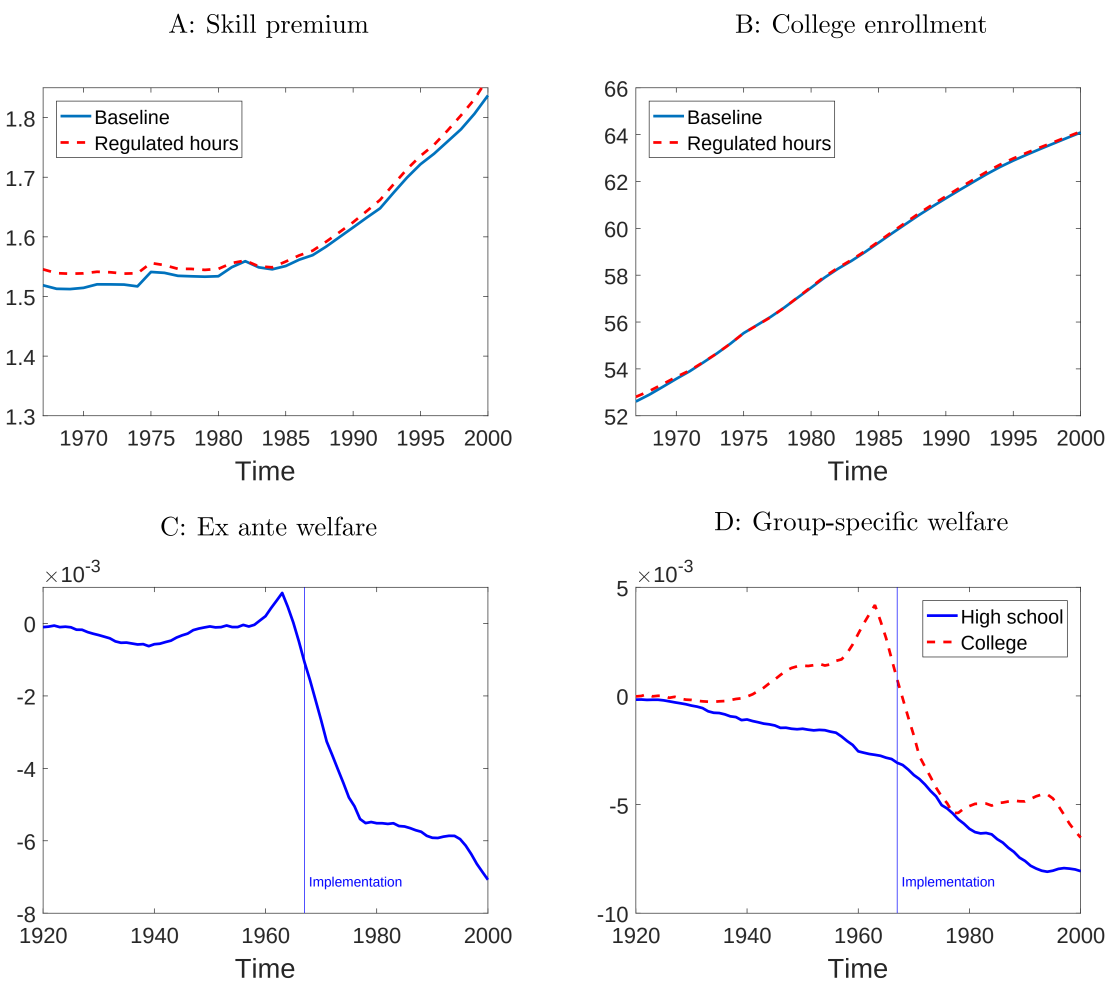

Figure 4: Consequences of hours regulation for education and wages.

Panel A shows the increase in the skill premium, which is defined as the wage of college graduates relative to the wage of those without a college degree. Panel B shows the increase in college enrollment. Panels C and D show the welfare effects of the regulation expressed in consumption equivalence units. Panel C shows the change in expected welfare for each age cohort before agents know their utility cost of education. Panel D shows welfare changes for the two education groups separately.

high school graduates is around 0.450, while the new regulation for both groups sets the maximum at 0.410.

In spite of the increase in the college premium, however, college enrollment does not increase. The reason is that the intertemporal labor supply effect, discussed in the previous session, is dampened by the workweek regulation. Without the regulation, working more hours would allow the high-skilled to reap additional benefits from education, thereby increasing education incentives. With the regulation, the educated cannot increase their hours as much, which lowers their education incentives.

This policy benefits the relatively well-off college-educated at the expense of the relatively poor high school graduates. This can also be seen in Panels C and D. Both show the welfare changes expressed as the compensating variation in consumption that each generation would need to be indifferent between living in the unregulated or the regulated economy. Positive numbers represent welfare gains and negative number welfare losses. Panel C shows that the ex ante welfare effects differ across generations, ranging from 0.1% for those born in 1963 to −0.7% for those born in 2000.

However, the ex ante perspective in Panel C masks some bigger effects for each skill group. Panel D shows the effects after the uncertainty about educational disutility is revealed for both groups, high school graduates and college graduates. All generations without a college degree lose out. For college graduates, there is an inter-generational welfare reversal. Essentially, those who are old before the regulation is enacted win out and those who are born later lose out. The reason is the interaction of the introduction of the regulation and the life-cycle of productivity of the high-skilled. Since productivity is hump-shaped over the life-cycle and declines for older workers, these workers never choose long hours, with or without regulation. Therefore the regulation does not affect them. The younger cohorts, however, would like to work more at the peak of their productivity but now are restricted. This negative effect from the restriction on hours dominates the positive effect from the higher skill premium for all those who enter the labor market after 1968. From an ex ante perspective, there are a few cohorts who benefit from the regulation. However, for most generations, the restriction on hours worked leads to a loss in welfare. The analysis thus identifies some counterproductive outcomes of policies such as the EU's Working Time Directive: if the goal is to help the uneducated and poor workers by setting a limit on working hours, the goal is partially missed, as the policy hurts these workers more than the well-educated workers in terms of income inequality, because of general equilibrium effects.

5.2 Limit on hours worked of low-skilled only

In this section, we analyse the effect of limiting the hours choice of the low-skilled only. There are two reasons for doing so. One is that regulation for the high-skilled is in many countries somewhat less stringent than for low-skilled agents (Messenger, 2006). The other is that we have seen that restricting the hours choice of the high-skilled hampers the general equilibrium effect of lowering the skill premium, to the detriment of the low-skilled. Therefore, we analyse a regulation limiting the hours choice of high-school graduates l^h, while leaving the one of college graduates l^c unconstrained,

$$l^h \leq l^{max}. \tag{13}$$

The results are shown in Figure 5. Panel A shows that now there is a small reduction in the skill premium compared to the unregulated case, opposite the impact of hours restriction on all workers seen in the previous section. Enrollment is again hardly affected, as in the case of hours restriction on all workers. The same mechanism operates. Now, it is only the unskilled who have to lower their labor supply, while the high-skilled are allowed to choose their labor supply freely. The resulting reduction in the supply of unskilled labor leads to the decrease in the skill premium.

The welfare changes in Panels C and D are now significantly smaller than before. The maximum fall in the consumption equivalent in Panel C of Figure 5 is about seven times lower than in Panel C of Figure 4. Moreover, as anticipated, the distributional effects are now significantly different. Whereas before, it was all generations of high-school graduates who lost out from the regulation, it is now college graduates who unambiguously lose out, because the skill premium is reduced. The fall of the skill premium also represents a relative gain for some unskilled workers: those who are relatively old when the hours restriction is introduced do not want to work many hours since their productivity is not as high as during their prime age, and thus benefit from the regulation. However, those high-school graduates who are born after 1973 also suffer welfare losses from the regulation, because the constraint on working hours operates and reduces their earnings potential.

6 Concluding remarks

Education is an important investment decision for individuals: it has a larger impact on lifetime earnings inequality than shocks during the working life (Huggett, Ventura, and Yaron, 2011) and provides better insurance than savings or working hours (Heathcote,

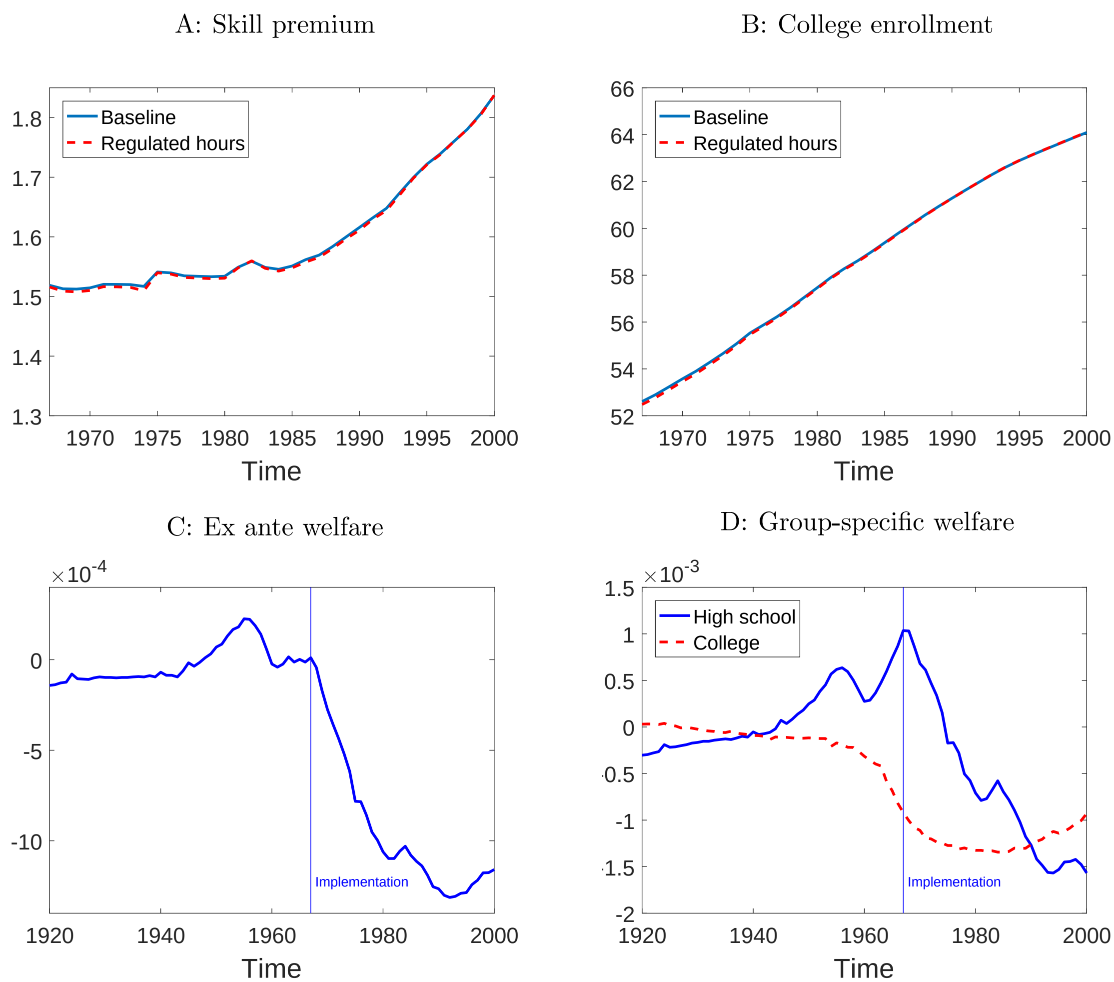

Figure 5: Consequences of low-skilled hours regulation for education and wages. Panel A shows the increase in the skill premium, which is defined as the wage of college graduates relative to the wage of those without a college degree. Panel B shows the increase in college enrollment. Panels C and D show the welfare effects of the regulation expressed in consumption equivalence units. Panel C shows the change in expected welfare for each age cohort before agents know their utility cost of education. Panel D shows welfare changes for the two education groups separately.

Storesletten, and Violante, 2010). Recent decades have seen a large increase in the skill premium and a steady increase in college enrollment. Understanding changes in education decisions and the forces behind these changes is crucial for a number of policy questions. For example, more high-skilled workers imply a larger tax base for income taxes; the quality of the labor force affects firms' investment decisions; and the political demand for redistribution depends on the distribution of pre-tax income.

The increase in the skill premium is a key factor behind changes in education, but it is not the only one. We show that relative variations in working hours matter, too. A standard neoclassical model with endogenous education and labor supply decisions where capital-skill complementarity and changes in the prices of capital equipment generate skill premium variations is able to simultaneously explain the increase in US male college enrollment and the change in the wage-hours correlation from negative to positive between 1967 and 2000. Since households are forward-looking, education and labor supply decisions are taken jointly. Education incentives are supported not only by an increase in the wage premium, but also by the possibility for educated workers to reap additional benefits by working longer hours. However, the longer working hours of high-skilled households will endogenously depress the skill premium. We show that the former effect is dominated by the latter: identical hours lead to an underestimate of the enrollment rate by about three percentage points, while exogenous factor prices lead to a fall in the skill premium by fifteen to thirty percentage points.

The effects of taxation and pensions on education incentives are relatively well understood. In settings with inelastic labor supply for instance, He and Liu (2008) show that progressive taxation reduces education incentives and Casarico and Devillanova (2008) show that redistributive pay-as-you-go pension systems have a similarly negative effect. Investigating whether these results hold with endogenous labor supply is left for future research.

We show that another policy instrument, restrictions on working hours, leads to partially counterproductive outcomes: if the goal is to promote the welfare of the most vulnerable segment of the working population, imposing a general restriction on working hours actually benefits old educated workers, while all uneducated and young educated workers lose out from the introduction of the regulation, because such a policy has very little effect on enrollment but leads to a higher skill premium. This is not the case if only the working hours for the low-skilled are limited. In that case, all generations of college-educated workers lose out but the early generations of high-school graduates actually gain. Nevertheless, from an ex ante welfare perspective, almost all generations lose out. We leave endogenizing the regulation within a political economy framework for future research. However, our finding that some groups sometimes gain at the expense of others points to the importance of taking imperfect substitutability of labor into account when analyzing labor regulations.

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A Appendix

A.1 Proofs

Proof (Lemma 2): Consider two households born at the same time v, one educated and one non-educated. For ease of reading, we drop the index v. Because households are born with no assets, the lifetime budget constraint is $\sum_{t=1}^{J} c_t^s / (1+r)^{t-1} = \sum_{t=1}^{J} w_t^s \varepsilon_t^s l_t^s / (1+r)^{t-1}$. From lemma 1 with a constant interest rate, $c_t^s = ((1+r)\beta)^{t-1} c_1^s$, so the lifetime budget constraint is

$$c_1^s = \frac{1}{\bar{\beta}} \sum_{t=1}^{J} \frac{w_t^s \varepsilon_t^s l_t^s}{(1+r)^{t-1}} \qquad \bar{\beta} \equiv \sum_{t=1}^{J} \beta^{t-1} = \frac{1-\beta^J}{1-\beta}.$$

Define $\kappa = c_1^c / c_1^h$. By the lemma 1, $(l_v^h(t)/l_v^c(t))^{\epsilon}(1+\alpha_t) = c_v^c(t)/c_v^h(t)$ is constant, so

$$l_t^c = \left(\frac{1+\alpha_t}{\kappa}\right)^{1/\epsilon} l_t^h. \tag{14}$$

Plugging the lifetime budget constraint, one has

$$\kappa = \frac{\sum_{t=1}^{J} \frac{(1+\alpha_t)^{1+1/\epsilon} w_t^h \varepsilon_t \left(\frac{1}{\kappa}\right)^{1/\epsilon} l_t^h}{(1+r)^{t-1}}}{\sum_{t=1}^{J} \frac{w_t^h \varepsilon_t l_t^h}{(1+r)^{t-1}}} \quad \Leftrightarrow \quad \kappa^{1+1/\epsilon} = \frac{\sum_{t=1}^{J} \frac{(1+\alpha_t)^{1+1/\epsilon} w_t^h \varepsilon_t l_t^h}{(1+r)^{t-1}}}{\sum_{t=1}^{J} \frac{w_t^h \varepsilon_t l_t^h}{(1+r)^{t-1}}}. \tag{15}$$

Assume first that the skill premium is constant over the lifetime of both households ($\alpha_t = \alpha$). From (15), one has $\kappa = 1+\alpha$, so, with (14):

$$l_t^c = l_t^h \qquad c_t^c = (1+\alpha)\, c_t^h.$$

Assume otherwise that the skill premium strictly increases at some point of the lifetime of both households and never decreases. Then $\alpha_{v+J} \geq \alpha_t \geq \alpha_v$ for all $t \in [v, v+J]$ and $\alpha_t > \alpha_v$ for $t \geq u$ for some index u. From (15) one has $(1+\alpha_v) < \kappa < (1+\alpha_{v+J})$. From (14) one has

$$l_v^c = \left(\frac{1+\alpha_v}{\kappa}\right)^{1/\epsilon} l_v^h < l_v^h \qquad l_{v+J}^c = \left(\frac{1+\alpha_{v+J}}{\kappa}\right)^{1/\epsilon} l_{v+J}^h > l_{v+J}^h.$$

Lemma 1 says that $(l_v^h(t)/l_v^c(t))^\epsilon(1+\alpha_t)$ is constant. Thus, when the skill premium α_t is constant (or increases) over periods of the life of the households, the ratio l_t^c/l_t^h is constant (or increases).

QED

Proof (Lemma 3): The sample correlation between two variables X and Y is negative (or positive) if and only if the regression line for the sample points on an XY-graph has a negative (or positive) slope. Here, we set wages on the X axis and hours on the Y axis. The proof is made by induction on the number of generations which experience the change in skill premium.

For one generation, the regression line at a given time period goes through two points exactly, (w_t^c, l_t^c) and $\left(w_t^h, l_t^h\right)$. By lemma 2, $l_t^c < l_t^h$ when $t < t_1$ and $l_t^c > l_t^h$ when $t \geq t_1$. Thus the regression line is negatively sloped before t_1 and positively sloped after t_1 (in fact, with correlation coefficients respectively -1 and $+1$).

By induction, assume the claim holds for generations $i \in \{1, ...n-1\}$: the regression line for $\left(w_t^{c,i}, l_t^{c,i}\right)$ and $\left(w_t^{h,i}, l_t^{h,i}\right)$ is negatively sloped for $t < t_1$. For generation $i = n$, $(w_t^{c,n}, l_t^{c,n})$ and $\left(w_t^{h,n}, l_t^{h,n}\right)$ have to satisfy two conditions. First, wages within education classes are the same, so $(w_t^{c,n}, l_t^{c,n})$ lies on the same vertical line as all other generations points $\left(w_t^{c,i}, l_t^{c,i}\right)$ and $\left(w_t^{h,n}, l_t^{h,n}\right)$ lies on the same vertical line as all other generations points $\left(w_t^{h,i}, l_t^{h,i}\right)$, for $i \in \{1, ...n-1\}$. Second, by lemma 2, $l_t^{c,n} < l_t^{h,n}$. In other words, the regression line for generation $i = n$ is negatively sloped. The regression line for generations $i \in \{1, ...n\}$ is a combination of the regression line for generations $i \in \{1, ...n-1\}$ and the regression line for generation $i = n$, both of which are negatively sloped and have their mass points on the same two vertical lines. Thus the regression line for generations $i \in \{1, ...n\}$ is also negatively sloped.

The same induction argument is applied for time periods $t \geq t_1$: the regression line for generations $i \in \{1, ...n\}$ is positively sloped.

Generations which do not experience the skill premium increase still influence the wage-hours correlation over the time horizon considered. The generation-specific regression lines for these generations are horizontal. Since their mass points (along the wage axis) lie on the same vertical lines, the all-generations regression line remains negative before t_1 and positive after t_1, as long as one or more generation which experienced the skill premium increase is alive.

QED.

A.2 Computational details

We provide a generic algorithm for computing a steady state, in a normalized system where the change of variable from K to $\tilde{K}$ has been performed (see Section 2.3). The extension to a transition path version is standard.

Algorithm: Steady-state computation with normalized system

1. Make guesses on capital stock $\tilde{K}$ and labor supply stocks S and U
2. Derive factor prices r, w^c and w^h with (11)
3. Compute value functions and household decision policy functions by solving backward the maximization system (4) subject to (3)
4. Using the fact that newborns start their life with zero assets, compute the path for simulation assets $a_{v,t}$, consumption $c_{v,t}$, education $s_{v,v}$ and labor supply $l_{v,t}$
5. Compute the resulting aggregate assets A with (7) and, from asset market clearing, derive simulated capital stock $\tilde{K}\prime = A$
6. Compute the resulting supplies of skilled labor S' and unskilled labor U' from (9)
7. Compare guesses $\tilde{K}$, S, U with simulated outcomes $\tilde{K}'$, S', U'; stop if they are close; otherwise update guesses and return to step 2.

Remarks: Further implementation details.

To solve the model numerically, initial and final steady states are needed. The economy is not in a steady state in 1967. In particular, the stock of skilled workers is not the same as the flow of new skilled workers entering the labor market: the college graduate rate in 1967 is larger than the fraction of the workforce which is college-educated (since the college graduation as increased steadily between 1950 and 1967). A similar comment holds for 2000. For a correct implementation of our model, we therefore choose a counterfactual initial steady state in 1900 and final steady state in 2070. With these choices, the model representations in 1967 and in 2000 are not steady states. Parameters are chosen so that the economic equilibrium in the model is close to the data in 1967. After 1967 and until 2000, the economic equilibrium are defined by the model dynamics, without the additional constraints brought about by steady states.

For implementation of counterfactual initial and final steady states, additional parameters are needed. The parameters are chosen so that they have no impact on equilibrium between 1967 and 2000. Data on the price of capital equipment, the driver of change in the model, ends in 2000. We assume that this price continues to grow for 20 years after 2000 (at the same yearly growth rate as the average between 1967 and 2000) and then remains constant for one generation, namely until 2070. To avoid jumps in household decisions around 1967, we also assume that the price of equipment starts to grow 25 years before 1967 (at half the yearly average growth rate between 1967 and 2000).

A.3 Calibration with constant hours

We calibrate the model with exogenous labor supply to the same moments as the baseline model except for hours worked and the wage-hours correlation, which are not defined in a model with exogenous and constant hours. The exogenous parameters remain the same as for the baseline model (see Table 1) and are not reported. Thus, we set the η, κ, and μ to match the college enrollment rate in 1967, the skill premium in 1967, and the skill premium in 2000.[14] Table 3 shows the resulting parameters.

Table 3: Calibrated model parameters with constant hours

	Parameter	Value
η	Mean in dist. education disutility	-0.700
κ	Variance in dist. education disutility	3.10
μ	Share for unskilled labor in production	0.417
λ	Share for capital in production	0.870

[14] Since we have only three targets, we leave the second parameter of the production function λ at its baseline value.