Online Appendix to Accompany *The Rise of the Added Worker Effect*

Jochen Mankart (Deutsche Bundesbank) Rigas Oikonomou (UC Louvain)

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This online appendix includes three sections. In Section A, we describe in detail the CPS data. In Section B, we present the proofs of the model.

A Data appendix

A.1 CPS: Description, Definitions and Filtering

The Current Population Survey (CPS) is a monthly survey of about 60,000 households (56,000 prior to 1996 and 50,000 prior to 2001), conducted jointly by the Census Bureau and the Bureau of Labor Statistics.[1] Survey questions cover employment, unemployment, earnings, hours of work, and a variety of demographic characteristics such as age, sex, race, marital status, and educational attainment. Although the CPS is not an explicit panel survey it does have a longitudinal component that allows us to construct the monthly labor market transitions in Section 2 of the paper. Specifically the design of the survey is such that the sample unit is interviewed for four consecutive months and then, after an eight-month rest period, interviewed again for the same four months one year later. Households in the sample are replaced on a rotating basis, with one-eighth of the households introduced to the sample each month. Given the structure of the survey we can match roughly three-quarters of the records across months.[2]

Using these matched records, we calculate the gross worker flows (for the aggregate and by gender age group and marital status) that we report in Section 2 of the paper. Our sample covers the period 1994 (January) 2014 (October). The flows are estimates of a Markov transition matrix where the three states are employment, unemployment and out of the labor force. We also broaden the definition of the labor market status variable when we explicitly consider non searchers, this gives a four by four matrix of transitions.[3]

We use the CPS classification rule to assign each member of a household to a labor market state. This rule is as follows: Employed agents are those who did (any) work for either pay or profit during the survey week. Unemployed are those who do not have a job, have actively looked for work in the month before the survey, and are currently available for work. "Actively looking" means that respondents have used one (or more) of the nine search methods considered by the CPS (6 methods prior to 1994) such as sending out resumes, responding to job adds, being enrolled with a public or private employment agency etc. Individuals who search "Passively" by attending a job training program or simply looking at adds are not considered as unemployed because these methods, according to the CPS, do not result in a sufficiently high arrival rate of job offers. The exception is workers on temporary layoff, i.e those workers who expect to be recalled by their previous employer. Those are counted as unemployed even if they do not search actively.

Finally, out of labor force are all agents who are neither employed nor unemployed (based on the above definitions). Amongst these we can (in the post 1994 period) separate individuals by reason of inactivity (for example, schooling, family responsibilities, disability, retirement etc) when we are interested in a finer selection of participants in this group. We obviously can also distinguish between individuals who are out of the labor force as non searchers.

Given this information we calculate the conditional probability that an agent who is in state i in the previous month (interview date) is in state j this month, where $(i, j) \in \{E, U, O\}$. We use the

[1] This is based on the data appendix of Mankart and Oikonmou (2015).

[2] Unfortunately, there is some sample attrition from individuals who abandon the survey (see for example Nagypál (2005) for a discussion of these issues).

[3] The survey allows us to identify non searchers in the post 1994 period, in previous years we can only identify an individual's labor market status as $\{E, U, O\}$, but not the distinction between individuals who want jobs but do not search and those who do not want jobs.

household weights provided by the CPS so that these objects are representative of the US population and we remove seasonal effects using a standard ratio to moving average approach (Shimer, 2012).

A.2 Transition matrices and AWE estimation

Monthly flows

Table 1: Added worker effect over time

	AWE
EU 1982	0.0357
	(0.0072)***
EU 1985	0.0593
	(0.0093)***
EU 1988	0.0562
	(0.0104)***
EU 1991	0.0554
	(0.0095)***
EU 1994	0.0473
	(0.0136)***
EU 1997	0.0588
	(0.0162)***
EU 2000	0.0796
	(0.0153)***
EU 2003	0.0764
	(0.0153)***
EU 2006	0.1302
	(0.0181)***
EU 2009	0.0776
	(0.0132)***
EU 2012	0.0949
	(0.0161)***
No of Kids	−0.00002
	(0.0027)
No of Kids ≤ 5	−0.0249
	(0.0005)***
Black	0.0539
	(0.0020)***
White	0.0110
	(0.01298)***
Educ 2	0.0131
	(0.0003)***
Educ 1	−0.0069
	(0.0003)***
Age_f	−0.0164
	(0.0029)***
Age_f^2	0.00043
	(0.0001)***
Age_f^3	$-3.9e-6$
	$(6.2e-7)$***
Age_m	−0.0012
	(0.0003)***
Age_m^2	miss
	(0.0001)***
Age_m^3	$1.4e-7$
	$(5.5e-8)$**
Const.	0.3375
	0.0357***
R^2	0.0067
No of obs.	893734

Note: This table shows estimates time-varying estimates of the added worker effect (AWE) from a linear probability model. The changes in the AWE are reflected by time dummies. Figure 1 in the paper is based on these estimates.

Table 2: Monthly Flow Rates: 1980s

	A: Married Men To			B: Married Women To		
From	E	U	O	E	U	O
E	0.984	0.011	0.005	0.939	0.011	0.050
U	0.293	0.639	0.069	0.212	0.506	0.282
O	0.103	0.065	0.832	0.048	0.021	0.932

Note: The table shows average monthly transition probabilities across the three labor market states: employment E, unemployment U and O for selected subgroups. Panels A and B show the flow rates for husbands and wives, respectively, while panel C shows the rates for household heads. See the online data appendix for further details on how the estimates are constructed.

Table 3: Monthly Flow Rates: 2000s

	A: Married Men To			B: Married Women To		
From	E	U	O	E	U	O
E	0.987	0.008	0.005	0.942	0.010	0.048
U	0.346	0.573	0.081	0.252	0.460	0.289
O	0.103	0.049	0.848	0.054	0.019	0.927

Note: The table shows average monthly transition probabilities across the three labor market states: employment E, unemployment U and O for selected subgroups. Panels A and B show the flow rates for husbands and wives, respectively, while panel C shows the rates for household heads. See the online data appendix for further details on how the estimates are constructed.

B Model Proofs

B.1 Value Functions and Derivations

The Bellman equations derived in text can be rewritten as follows:

$$W_{EE} = u(w_m + w_f) - \omega + \beta(1-s)^2 \max\{W_{EE}, W_{EU}, W_{EO}\} +$$
$$\beta s(1-s)\max\{W_{EU}, W_{EO}\} + \beta s(1-s)\max\{W_{UE}, W_{UU}, W_{UO}\} + \beta s^2 \max\{W_{UU}, W_{UO}\}$$

$$W_{EU} = u(w_m) - \omega\kappa + \beta(1-s)p_{U,f}\max\{W_{EE}, W_{EU}, W_{EO}\} +$$
$$\beta(1-p_{U,f})(1-s)\max\{W_{EU}, W_{EO}\} + \beta s p_{U,f}\max\{W_{UE}, W_{UU}, W_{UO}\}$$
$$+\beta(1-p_{U,f})s\max\{W_{UU}, W_{UO}\}$$

$$W_{EO} = u(w_m) + \beta(1-s)p_{O,f}\max\{W_{EE}, W_{EU}, W_{EO}\} +$$
$$\beta(1-p_{O,f})(1-s)\max\{W_{EU}, W_{EO}\} + \beta s p_{O,f}\max\{W_{UE}, W_{UU}, W_{UO}\}$$
$$+\beta(1-p_{O,f})s\max\{W_{UU}, W_{UO}\}$$
$$W_{UE} = u(b + w_f) - \omega + \beta[p_{U,m}(1-s)\max\{W_{EE}, W_{EU}, W_{EO}\} + p_{U,m}s\max\{W_{EU}, W_{EO}\} +$$
$$(1-p_{U,m})(1-s)\max\{W_{UE}, W_{UU}, W_{UO}\} + (1-p_{U,m})s\max\{W_{UU}, W_{UO}]$$

$$W_{UU} = u(b) - \omega\kappa + \beta p_{U,m}p_{U,f}\max\{W_{EE}, W_{EU}, W_{EO}\} +$$
$$\beta(1-p_{U,f})p_{U,m}\max\{W_{EU}, W_{EO}\} + \beta(1-p_{U,m})p_{U,f}\max\{W_{UE}, W_{UU}, W_{UO}\}$$
$$+\beta(1-p_{U,f})(1-p_{U,m)}\max\{W_{UU}, W_{UO}\}$$

$$W_{UO} = u(b) + \beta p_{U,m}p_{O,f}\max\{W_{EE}, W_{EU}, W_{EO}\} +$$
$$\beta(1-p_{O,f})p_{U,m}\max\{W_{EU}, W_{EO}\} + \beta(1-p_{U,m})p_{O,f}\max\{W_{UE}, W_{UU}, W_{UO}\}$$
$$+\beta(1-p_{O,f})(1-p_{U,m)}\max\{W_{UU}, W_{UO}\}$$

For these expressions and from Figure 2 (policy functions) we now derive the ωs.

The expression for ω_4 Consider first the case where $\omega = \omega_4$. In this case we have that: $W_{UE} = W_{U0}$. Moreover, from the graph we can infer the following: i) $\max\{W_{EE}, W_{EU}, W_{EO}\} = W_{EO}$ ii) $\max\{W_{EU}, W_{EO}\} = W_{EO}$ iii) $\max\{W_{UE}, W_{UU}, W_{UO}\} = W_{UO} = W_{UE}$. $\max\{W_{UU}, W_{UO}\} = W_{UO}$. Given these properties the Bellman equations become:

$$W_{UO} = u(b) + \beta p_{U,m}p_{O,f}W_{EO} + \beta(1-p_{N,f})p_{U,m}W_{EO} + \beta(1-p_{U,m})p_{O,f}W_{UN} + \beta(1-p_{O,f})(1-p_{U,m)}W_{UO}$$
$$W_{UE} = u(b + w_f) - \omega + \beta(1-s)p_{U,m}W_{EO} + \beta(1-p_{U,m})(1-s)W_{UO} + \beta s p_{U,m}W_{UE} + \beta(1-p_{U,m})sW_{UO}$$

It is simple to show that $W_{UE} - W_{UO} = 0 \rightarrow u(b + w_f) - \omega_4 - u(b) = 0$. This gives the value for ω_4.

The expression for ω_3 Assume now that $W_{EE}(\omega_3) = W_{EO}(\omega_3)$. From Figure 2 (after simplifying the expressions which involve the max operator in the value functions) we can write:

$$(1) \qquad W_{EE}(\omega_3) - W_{EO}(\omega_3) = 0 \rightarrow u(\sum_g w_g) - \omega_3 - u(w_m) + \beta s(1 - s - p_{O,f})(W_{UE} - W_{UO})$$

Moreover, we can illustrate that

$$W_{UE}(\omega_3) - W_{UO}(\omega_3) = 0 \rightarrow u(b + w_f) - \omega_3 - u(b) - \beta p_{O,f}(1 - p_{U,m})(W_{UE}(\omega_3) - W_{UO}(\omega_3))$$
$$(W_{UE}(\omega_3) - W_{UO}(\omega_3)) = \frac{u(b + w_f) - \omega_3 - u(b)}{1 + \beta p_{O,f}(1 - p_{U,m})}$$

Making use of this expression we can write (1)

$$(2) \qquad \omega_3[1 + \beta \frac{s(1 - s - p_{O,f})}{1 + \beta(1 - p_{U,m})p_{O,f}}] = u(\sum_g w_g) - u(w_m) + \beta s(1 - s - p_{O,f}) \frac{u(b + w_f) - u(b)}{1 + \beta p_{O,f}(1 - p_{U,m})}$$

as was claimed in text.

The expressions for ω_1 and ω_2 Now to define ω_1 and ω_2 we have:

$$W_{EU}(\omega_1) = W_{EO}(\omega_1) \quad and \quad W_{UO}(\omega_2) = W_{UU}(\omega_2)$$

Lets begin with ω_1. From Figure 2 we have that

$$W_{EU}(\omega_1) - W_{EO}(\omega_1) = 0 \rightarrow$$
$$\beta(1 - s)(p_{U,f} - p_{O,f}) \underbrace{(W_{EE} - W_{EU})}_{>0} + \beta s(p_{U,f} - p_{O,f}) \underbrace{(W_{UE} - W_{UU})}_{>0} - \kappa\omega_1 = 0$$

defines ω_1. To recover the terms $(W_{EE} - W_{EO})$ and $(W_{UE} - W_{UO})$ we use the Bellman equations for these objects. We can easily show that:

$$W_{EE}(\omega_1) - W_{EU}(\omega_1) = u(w_m + w_f) - \omega_1(1 - \kappa) - u(w_m) +$$
$$\beta(1 - s)(1 - s - p_{U,f})(W_{EE} - W_{EU}) + \beta s(1 - s - p_{U,f})(W_{UE} - W_{UO})$$

$$W_{UE}(\omega_1) - W_{UO}(\omega_1) = u(b + w_f) - \omega_1(1 - \kappa) - u(b) + \beta p_{U,m}(1 - s - p_{U,f})(W_{EE} - W_{EO})$$
$$+ \beta(1 - p_{U,m})(1 - s - p_{U,f})(W_{UE} - W_{UO})$$

We need to solve a system of two equations to find the capital gains. Solving the system gives:

$$W_{EE}(\omega_1) - W_{EU}(\omega_1) = \frac{(1-\beta(1-p_{U,m})(1-s-p_{U,f}))\xi_1 + \beta s(1-s-p_{U,f})\xi_2}{\Delta}$$
$$W_{UE}(\omega_1) - W_{UO}(\omega_1) = \frac{(1-\beta(1-s)(1-s-p_{U,f}))\xi_2 + \beta p_{U,m}(1-s-p_{U,f})\xi_1}{\Delta}$$

where $\xi_1 = u(w_m + w_f) - \omega_1(1-\kappa) - u(w_m)$, $\xi_2 = u(b + w_f) - \omega_1(1-\kappa) - u(b)$ and $\Delta = [1-\beta(1-s-p_{U,f})][1-\beta(1-s-p_{U,f})(1-s-p_{U,m})]$.

With the above it is simple to obtain the expression of ω_1 in text. Put together all the terms multiplying ξ_1 gives:

$$\frac{\beta(p_{U,f}-p_{O,f})}{\Delta}[1-s-\beta(1-p_{U,m})(1-s-p_{U,f}) + s\beta p_{U,m}(1-s-p_{U,f}] =$$
$$\frac{\beta(p_{U,f}-p_{O,f})}{\Delta}[1-s-\beta(1-p_{U,m}-s)(1-s-p_{U,f})]$$

and the terms multiplying ξ_2 may be written as:

$$\frac{\beta s(p_{U,f}-p_{O,f})}{\Delta}[\beta(1-s)(1-s-p_{U,f}) + 1 - \beta(1-s)(1-s-p_{U,f})] = \frac{\beta s(p_{U,f}-p_{O,f})}{\Delta}$$

Therefore we have that:

$$\frac{\beta s(p_{U,f}-p_{O,f})}{\Delta}\xi_2 + \frac{\beta(p_{U,f}-p_{O,f})}{\Delta}[1-s-\beta(1-p_{U,m}-s)(1-s-p_{U,f})]\xi_1 - \kappa\omega_1 = 0$$

Now let $\widetilde{\xi}_1 = u(w_m + w_f) - u(w_m)$, $\widetilde{\xi}_2 = u(b + w_f) - u(b)$. Taking all the terms multiplying ω_1 on the RHS of the previous equation we get:

$$\frac{\beta s(p_{U,f}-p_{O,f})}{\Delta}\xi_2 + \frac{\beta(p_{U,f}-p_{O,f})}{\Delta}[1-s-\beta(1-p_{U,m}-s)(1-s-p_{U,f})]\xi_1 =$$
$$\omega_1[\kappa + (1-\kappa)\frac{\beta(p_{U,f}-p_{O,f})}{1-\beta(1-s-p_{U,f})}]$$

We now apply the same procedure to recover ω_2.

$$W_{UU}(\omega_2) - W_{UO}(\omega_2) = 0 \rightarrow$$
$$\beta p_{U,m}(p_{U,f}-p_{O,f})\underbrace{(W_{EE}-W_{EO})}_{>0} + \beta(1-p_{U,m})(p_{U,f}-p_{O,f})\underbrace{(W_{UE}-W_{UO})}_{>0} - \kappa\omega_2 = 0$$

To recover the terms $(W_{EE} - W_{EO})$ and $(W_{UE} - W_{UO})$ we use the Bellman equations for these objects. We can easily show that:

$$W_{EE}(\omega_2) - W_{EO}(\omega_2) = u(w_m + w_f) - \omega_2 - u(w_m) +$$
$$\beta(1-s)(1-s-p_{O,f})(W_{EE} - W_{EO}) + \beta s(1-s-p_{O,f})(W_{UE} - W_{UO})$$

$$W_{UE}(\omega_2) - W_{UO}(\omega_2) = u(b + w_f) - \omega_2 - u(b) + \beta p_{U,m}(1-s-p_{O,f})(W_{EE} - W_{EO})$$
$$+\beta(1-p_{U,m})(1-s-p_{O,f})(W_{UE} - W_{UO})$$

We need to solve a system of two equations to find the capital gains. Solving the system gives:

$$W_{EE}(\omega_2) - W_{EO}(\omega_2) = \frac{(1-\beta(1-p_{U,m})(1-s-p_{U,f}))\xi_3 + \beta s(1-s-p_{U,f})\xi_4}{\Delta}$$
$$W_{EE}(\omega_2) - W_{EO}(\omega_2) = \frac{(1-\beta(1-s)(1-s-p_{U,f}))\xi_4 + \beta p_{U,m}(1-s-p_{U,f})\xi_3}{\Delta}$$

where $\xi_3 = u(w_m + w_f) - \omega_2 - u(w_m)$, $\xi_4 = u(b + w_f) - \omega_2 - u(b)$ and $\Delta = [1 - \beta(1-s-p_{O,f})][1 - \beta(1-s-p_{N,f})(1-s-p_{U,m})]$.

We therefore have:

$$\frac{\beta p_{U,m}(p_{U,f} - p_{O,f})}{\Delta}\xi_3 + \frac{\beta(1-p_{U,m})(p_{U,f} - p_{O,f})}{\Delta}[1 - \beta(1-s-p_{U,f})]\xi_4 = \kappa\omega_2$$

Therefore:

$$\omega_2[\kappa + \frac{1 - \beta(1-p_{U,m})(1-s-p_{O,f})]}{\Delta}] =$$
$$\frac{\beta p_{U,m}(p_{U,f} - p_{O,f})}{\Delta}\widetilde{\xi_3} + \frac{\beta(1-p_{U,m})(p_{U,f} - p_{O,f})}{\Delta}[1 - \beta(1-s-p_{U,f})]\widetilde{\xi_4}$$

These analytical expressions can be utilized to calibrate the model.

References

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