Heterogeneity in Expectations and House Price Dynamics*

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Abstract

Expectations are central for housing decisions and heterogeneity in expectations is a robust feature of survey data. We study the implications of heterogeneity in house price growth expectations for the level of house prices. We feed the joint empirical distributions of income, wealth and expectations into a calibrated heterogeneous agents housing model. We find that eliminating heterogeneity in house price growth expectations would raise average house prices and amplify house price fluctuations thereby reducing the fit of the model. Without heterogeneity, average house prices would be about 11 percent higher and the boom-bust cycle would be about 41 percent larger.

JEL Classification: D14,D84,D31,E21,E30,G21, R21

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1 Introduction

House price movements are often partly attributed to movements in expectations. For example, much of the recent literature on macroeconomics and housing is motivated by the international housing boom-bust episode of the early 2000s; and while Justiniano, Primiceri, and Tambalotti (2019) and Favilukis, Ludvigson, and Van Nieuwerburgh (2017) find that shocks to credit conditions were the key driving factor, Landvoigt (2017) and Kaplan, Mitman, and Violante (2020) stress the role played by beliefs about future house prices. Furthermore, several papers have been able to identify a causal link from housing market expectations to housing-related decisions at the individual level, either by exploiting data on social networks (Bailey, Cao, Kuchler, and Stroebel 2018; Bailey, Dávila, Kuchler, and Stroebel 2019) or by conducting information provision experiments (Armona, Fuster, and Zafar 2019; Bottan and Perez-Truglia 2024). One of the most striking features of survey data on beliefs about future house prices is the large heterogeneity in house price growth expectations (Kuchler, Piazzesi, and Stroebel 2023). Several papers investigate the sources of this heterogeneity (e.g. Kuchler and Zafar 2019; Fuster, Perez-Truglia, Wiederholt, and Zafar 2022). The question we address in this paper is how this heterogeneity in house price growth expectations across households affects the level and the dynamics of house prices.

Our paper contributes to this literature by investigating how measured subjective house price growth expectations and their heterogeneity influence equilibrium house prices in a structural housing model. As the main difference to the existing literature, we explore explicit measures of expectations at the individual household level and do not rely on implicit measures derived from a particular economic model. We find that the house price growth expectations elicited in the survey data and their heterogeneity play an important role for the level and the dynamics of equilibrium house prices.

More specifically, to address the question, we solve a life-cycle model of the housing market with subjective house price growth expectations. Households choose consumption, holdings of a riskless asset, as well as the extensive margin of housing (i.e., whether to rent or own) and the intensive margin of housing (i.e., the size of the rented home or the size of the purchased home). In the dynamic programming problem of an individual household, the state variables are: age, income, the beginning-of-period financial wealth, the beginning-of-period housing wealth, and expectations of future house prices. We compute the choices of an individual household with the policy function and the state variables taken from the data—exploiting a survey that includes in each year for each household information on age, income, different components of wealth, and expectations—and we compute the sequence of market clearing prices. Finally, we ask: What would the time series of market clearing house prices had been, if we had abstracted from the observed heterogeneity in house price growth expectations?

We find that abstracting from the observed heterogeneity in house price growth expectations yields a higher average level of house prices and a larger amplitude of house price fluctuations. Put differently, the heterogeneity in house price growth expectations reduces the average level of house prices and reduces the amplitude of house price fluctuations.

Many features of the housing market could be driving these results. In the housing market, expectations correlate to some extent with other variables affecting housing demand, such as age. By abstracting from the heterogeneity in house price growth expectations, one is to some extent reallocating expectations to households with systematically different demographics, which could affect aggregate housing demand. By contrast, taking into account the expectations heterogeneity in the data, one allows for the possibility of a correlation between expectations and demographics. Furthermore, for a given age, income, and wealth, individual housing demand is likely to be highly non-linear in expectations of future house price growth because of various constraints: the debt-to-income (DTI) constraint, the loan-to-value (LTV) constraint, and the no short-selling constraint. Our model features all these constraints.

To understand which mechanisms are driving our main results, we perform two exercises in the calibrated model. First, we visualize individual housing demand as a function of the house price growth expectation, after integrating over all other state variables in the model. It turns out that individual housing demand is a convex-concave function of the house price growth expectation. At a very low house price growth expectation, households are insensitive to a small variation in the expectation because they are renting anyway (e.g., they are postponing buying). At an intermediate range of the house price growth expectation, the size of the purchased home is strongly increasing in the expectation of future house price growth. At a high house price growth expectation, the size of the purchased home becomes fairly insensitive to the expectation because of binding constraints (and in particular due to the DTI constraint). This could explain our main finding about the amplitude of house price fluctuations, because the convexity at the bottom implies that expectations heterogeneity drives the house price up at low average house price growth expectations, while the concavity at the top implies that expectations heterogeneity drives the house price down at high average house price growth expectations. Furthermore, this convex-concave shape of the individual housing demand function could also explain our main finding about the average level of house prices, since in our data set we have more boom years than bust years. Second, to confirm the importance of the DTI constraint for the effects of expectations heterogeneity, we resolve the model with the DTI constraint switched off. We indeed find that the sequence of house prices in the heterogeneous expectations variant of the model and in the homogeneous expectations variant of the model become much more similar.

We build on a growing literature arguing that a natural approach to learn about agents' expectations is to elicit expectations (see Manski (2004) on the advantages of collecting survey data on expectations; and see Section 1 in Kuchler, Piazzesi, and Stroebel (2023) for a description of existing surveys of housing market expectations). We use data from the De Nederlandsche Bank (DNB) Household Survey, which is, to the best of our knowledge, the only survey that combines the following two features: first, this representative survey contains questions on expectations of future house prices as well as detailed questions on income and wealth; and second, the survey has been fielded for a long time. Because of these two features, we can use the *joint distribution* of income, wealth, and expectations of future house prices as an input in the model; and we can investigate the effects of expectations heterogeneity on the level of house prices over all phases of the boom-bust-rebound cycle in house prices that started in the early 2000s.

To solve the calibrated model, we implement the *temporary equilibrium with directly measured expectations* approach suggested by Piazzesi and Schneider (2016) in their Section 10.[1]

Related Literature The paper is related to several strands of literature. First, the paper is related to the literature on structural models of the housing market (e.g., Favilukis, Ludvigson, and Van Nieuwerburgh 2017; Kaplan, Mitman, and Violante 2020). A very small number of papers in this literature allow for heterogeneity in house price expectations, even though the heterogeneity in house price expectations is such a robust feature of survey data. Piazzesi and Schneider (2009) study the responses to qualitative questions in the Michigan Survey of Consumers (MSC) and document that the fraction of households who said now is a good time to buy a house because house prices will rise further ("momentum households") rose from about 10 percent in 2003 to over 20 percent in 2005. To show that this change can have a large price impact, they consider a simple search model of the housing market with three types of households: happy owners, unhappy owners (who have been hit by a preference shock since buying), and renters. They consider a one-time unanticipated shock that makes all renters—assumed to be less than 3 percent of the population—optimistic about future house prices. Their main insight is that a small fraction of optimists can have a large price impact in this search model. Burnside, Eichenbaum, and Rebelo (2016) incorporate an epidemiological model of belief formation into a matching model of the sort considered by Piazzesi and Schneider (2009). In terms of beliefs, there are three types of agents: optimistic agents expect an improvement in fundamentals, skeptical and vulnerable agents do not expect fundamentals to improve, and vulnerable agents have a more diffuse prior than optimistic and skeptical agents. When two agents meet, the agent with the lower-entropy prior "infects" the agent with the higher-entropy prior with a probability that is decreasing in the ratio of the entropies of the two priors. They compute the transition to a medium-run equilibrium, where the

[1] Piazzesi and Schneider (2016) base this notion on Grandmont (1977).

entire population has converged to the view of the agent with the tightest prior but uncertainty about fundamentals has not been resolved yet. The model generates a boom-bust cycle in house prices along the transition, if skeptical agents are the agents with the tightest prior and initial conditions are such that the fraction of optimistic agents rises for a while before converging to zero. By contrast, the model features a boom that is not followed by a bust, if optimistic agents have the tightest prior and initial conditions are such that the fraction of optimistic agents rises monotonically. In the long run (i.e., after uncertainty resolution), house prices depend on whether optimistic or skeptical agents happen to be correct. In these models, all agents have linear utility, they can purchase a home of a single size, they do not face any credit constraints, and they have sufficient life-time income to purchase a home. Hence, a household's decision whether to purchase a home depends only on the household's utility of owning versus renting, the household's expectation of future house prices, and if the household is matched with a seller. Moreover, the main channels that we are emphasizing are absent due to the absence of a DTI and a LTV constraint.

Second, a recent literature studies the causes of expectations heterogeneity and has identified determinants such as own recent personal experiences (Kuchler and Zafar 2019), experiences of friends (Bailey, Cao, Kuchler, and Stroebel 2018), the choice to look at different pieces of information (Fuster, Perez-Truglia, Wiederholt, and Zafar 2022), ownership status (Kindermann, Le Blanc, Piazzesi, and Schneider 2024), local conditions (Kiesl-Reiter, Lührmann, Shaw, and Winter 2024), and heterogeneity in priors about long-run house price growth (Li, Van Nieuwerburgh, and Renxuan 2023). Furthermore, a few papers have managed to provide causal evidence on the link between expectations and individual housing market behavior by exploiting plausibly exogenous variation in house price growth expectations across individuals. These papers have found large effects of individuals' housing market expectations on individuals' housing market behavior: the decision whether to rent or own, the square footage of the home bought, and the willingness to pay for a given home (Bailey, Cao, Kuchler, and Stroebel 2018), the mortgage leverage choice (Bailey, Dávila, Kuchler, and Stroebel 2019), and the selling probability (Bottan and Perez-Truglia 2024). We study the consequences of the heterogeneity in house price growth expectations for the level of house prices.

Third, the paper is related to a nascent literature studying heterogeneous agent models with expectations of future prices that are consistent with survey data on expectations (e.g. Broer, Kohlhas, Mitman, and Schlafmann 2022). Survey data on expectations is rich and has many robust features (e.g., large heterogeneity in expectations, predictability of average forecast errors, predictability of individual forecast errors). Solving state-of-the-art heterogeneous agent models can be challenging, despite significant advances in solution methods. We therefore think that an important step forward is to understand *which features* of the survey data on expectations are important to match in the heterogeneous agent models. Since, in these models, it is the combination of households who are borrowing constrained and households who are currently not borrowing constrained that generates a realistic average marginal propensity to consume (e.g., in the benchmark calibration of Kaplan, Mitman, and Violante (2020), the share of households at the zero kink of the budget constraint equals about 30 percent; see their Table 5), it seems natural to conjecture that heterogeneity in expectations can have a sizeable effect on market outcomes, because of the non-linearity that a binding borrowing constraint induces in an agent's policy function. Moreover, it seems natural to investigate this conjecture in a model with housing, because of the high share of households with low liquid wealth but positive illiquid wealth (so called wealthy hand-to-mouth households) in the data. Indeed, we find that the extent of disagreement about future house price growth observed in the data significantly decreases (increases) equilibrium house prices, when the average house price growth expectation is high (low).

Fourth, our paper is related to the literature on belief disagreement and financial speculation (i.e., the trading of financial assets by investors with heterogeneous beliefs). It is well known in this literature that, with short-selling constraints, belief disagreement can generate overvaluation of an asset. See Simsek (2021) for a stylized macroeconomic model with financial speculation and for a review of this literature. This mechanism is present in our structural model of the housing market at low average house price growth expectations, because short selling real estate is not possible.

The paper is organized as follows. Section 2 presents a simple two-period model to illustrate how the combination of the short-selling constraint on housing and the debt-to-income constraint generates a housing demand function that is convex-concave in house price growth expectations. We also show how this feature of the housing demand function shapes the effect of expectations heterogeneity on the house price, at different levels of the average house price growth expectation. Section 3 introduces our data on house price growth expectations, income, wealth, and housing decisions. Section 4 develops a quantitative, life-cycle, housing model with subjective house price growth expectations. Section 5 defines equilibrium. Section 6 describes the choice of parameters. Section 7 shows how heterogeneity in house price growth expectations affects the market price for housing in the quantitative model. Section 8 concludes.

2 Housing Demand in a Two-Period Model

We develop a simple two-period lived households model to illustrate how heterogeneous house price growth expectations about prices in period 1 affect equilibrium prices in period 0. The main purpose of this simplified model is to develop intuition for the key mechanisms behind our quantitative findings. We show that with a short-selling constraint on houses and heterogeneous house price growth expectations, the equilibrium period 0 house price is driven up relative to a model with homogeneous expectations, if average house price growth expectations are low. In contrast, a debt-to-income constraint drives house prices down with heterogeneous expectations relative to homogeneous expectations, if average house price growth expectations are relatively high. A sequence of such two-period lived households would then imply that expectations driven fluctuations are lower in a model with heterogeneous than with homogeneous expectations.

2.1 Model Setup

Consider a two-period lived household i with preferences over consumption c_j in the two periods of life $j \in \{0, 1\}$ (where we drop the household index i unless needed)

$$u(c_0, c_1) = \ln(c_0) + \beta \ln(c_1),$$

where β is the discount factor.

The household is endowed with some initial assets $a_0 \geq 0$ and earns a fixed exogenous income of y in both periods. Households may invest in financial assets or housing. Housing is subject to a no short-selling constraint $h_1 \geq 0$, while financial assets are subject to a debt-to-income (DTI) constraint $a_1 \geq -\gamma y$ with $\gamma < 1$.[2] Consumption is the numeraire good and houses are traded in the initial period at price p_0. Since there is no bequest motive, the household will liquidate all assets and consume everything in period 1. For an inter-temporal price $q \leq 1$ of financial assets, the budget constraints in the two periods of life are

$$c_0 + qa_1 + p_0 h_1 \leq y + a_0, \qquad c_1 \leq y + a_1 + p_1 h_1.$$

Each household has some expectation of future house prices and believes that there is no uncertainty about future house prices. Let p_1^i denote household i's period-0 expectation of the house price in period 1.

Finally, there is an exogenous supply of houses H in the economy in the initial period, which, without loss of generality, we normalize to $H = 1$.

2.2 Analysis

Given the simple structure of this two-asset model, each household's portfolio decision depends only on the expected returns on housing and the financial asset, initial wealth, incomes, and the constraints. The short-selling constraint on housing and the debt-to-income constraint imply that

[2] In our quantitative model of Section 4, there will also be a loan-to-value constraint and housing will be part of the utility function.

there are four relevant cases:

$$1.)\, a_1 = -\gamma y, h_1 = 0, \quad 2.)\, a_1 > -\gamma y, h_1 = 0, \quad 3.)\, a_1 = -\gamma y, h_1 > 0, \quad 4.)\, a_1 > -\gamma y, h_1 > 0,$$

for which we derive the consumption, savings and housing choices.[3] Here we show only individual housing demand, which for individual gross house price growth expectation $\Delta P_1^i \equiv \frac{p_1^i}{p_0}$ and financial asset return $R \equiv \frac{1}{q}$, is

$$h_1 = \begin{cases} 0 & \text{if} \quad \Delta P_1^i \leq R \quad \vee \\ & \qquad a_0 \leq \left[\frac{(1-\gamma)-(1+q\gamma)\beta\Delta P_1^i}{\beta\Delta P_1^i}\right] y \\ \frac{1}{p_0}\frac{1}{1+\beta}\left[\beta(a_0 + y + \gamma q y) - \frac{1}{\Delta P_1^i}(1-\gamma)y\right] & \text{if} \quad \Delta P_1^i > R \quad \wedge \\ & \qquad a_0 > \left[\frac{(1-\gamma)-(1+q\gamma)\beta\Delta P_1^i}{\beta\Delta P_1^i}\right] y. \end{cases} \tag{1}$$

The first line shows that housing demand is zero for households with sufficiently pessimistic house price growth expectations, where sufficiently pessimistic means that the household expects gross house price growth to be lower than the gross return on the financial asset, i.e., $\Delta P_1^i \leq R$.[4] Individual housing demand is also zero in case the household is relatively asset poor. This is the second condition in the first line.

The second line shows that housing demand is positive for households who are the combination of sufficiently optimistic and relatively wealthy. The equation also shows that, in this case, individual housing demand is concave in the house price growth expectation. This is a result of the interplay of the utility function and the debt-to-income constraint. A household that expects $\Delta P_1^i > R$ would like to borrow as much as possible to invest into housing since he effectively expects an arbitrage opportunity. Due to the debt-to-income constraint, borrowing is limited and any additional housing investment must be financed by lower consumption which gets ever more costly in utility terms.[5]

Thus, our simple model setup yields the following housing demand function

$$h_1\left(p_0, \Delta P_1^i\right) = \max\left\{0, \frac{1}{p_0}\left(\phi - \psi\frac{1}{\Delta P_1^i}\right)\right\}, \tag{2}$$

where $\phi \equiv \frac{\beta}{1+\beta}\left(a_0 + (1+q\gamma)y\right)$, and $\psi \equiv \frac{1-\gamma}{1+\beta}y$. Equation (2) shows that housing demand is an increasing function in the individual house price growth expectation ΔP_1^i and a decreasing

[3] The details of these derivations are provided in the Supplementary Appendix.

[4] For simplicity, we assume that if the two returns are equal, the household invests only in the financial asset.

[5] Concavity is apparent in equation (1) because ΔP_1^i enters the denominator with a negative sign, implying $\frac{\partial h}{\partial(\Delta P_1^i)} > 0$ and $\frac{\partial^2 h}{\partial(\Delta P_1^i)^2} < 0$.

function of the current period market price p_0. The max operator implies that housing demand has a convex region for low house price growth expectations. The expression inside the max operator implies a concave region for higher house price growth expectations. Thus, housing demand features a convex-concave schedule.

Given a distribution $\Phi(\Delta P_1^i)$ of house price growth expectations such that $\int_R^\infty d\Phi(\Delta P_1^i) > 0$ and the assumed exogenous supply of houses of $H = 1$, the equilibrium price $p_0 > 0$ in the housing market is thus

$$p_0 = \int \max\left\{0, \phi - \psi \frac{1}{\Delta P_1^i}\right\} d\Phi(\Delta P_1^i).$$

2.3 An Illustrative Example

The important insight from equation (2) is the convex-concave schedule of housing demand in the house price growth expectation. We now develop an example to illustrate how this feature of the demand schedule may give rise to house price dynamics such that in a "regime" with $l$ow average house price growth expectations the equilibrium house price in a "scenario" with homogeneous house price growth expectations is below the equilibrium price of a scenario with heterogenous expectations, and vice versa in a regime with $h$igh average house price growth expectations. An implication is that the amplitude of house price movements is higher in the economy with homogeneous house price growth expectations.

To derive this result, we assume that within each expectations regime, there are two degenerate expected house price distribution scenarios. In the homogeneous expectations scenario, house price growth expectations for all individuals are given by $\bar{\Delta}P_1^h > \bar{\Delta}P_1^l$, respectively, where $\bar{\Delta}P_1^j = \int \Delta P_1^i d\Phi^j(\Delta P_1^i)$. In the heterogeneous expectations scenario, we assume that a fraction $\frac{1}{2}$ in the population holds low and a fraction $\frac{1}{2}$ holds high house price growth expectations relative to the respective mean expectations $\bar{\Delta}P_1^j$, $j \in \{l, h\}$. We parameterize these expectations by a symmetric spread κ such that heterogeneous expectations in the respective regime are given by $\left[\bar{\Delta}P_1^j - \kappa, \bar{\Delta}P_1^j + \kappa; \frac{1}{2}, \frac{1}{2}\right]$, for $j \in \{l, h\}$.

If $\bar{\Delta}P_1^l$ is only somewhat larger then R, housing demand with homogeneous expectations will be relatively small. In such a case, the corresponding heterogeneous expectations case will feature higher demand and therefore a higher price. The additional demand of the 50% of households with expectations $\bar{\Delta}P_1^l + \kappa$ will more than compensate the zero demand from the households with expectations $\bar{\Delta}P_1^l - \kappa < R$ for whom the short-selling constraint is binding. Such a case is shown in the illustrative example in Panel (a) of Figure 1, where $\bar{\Delta}P_1^l = 1.1$ and $R = 1$. For a sufficiently large spread, demand will be higher under heterogeneous expectations.

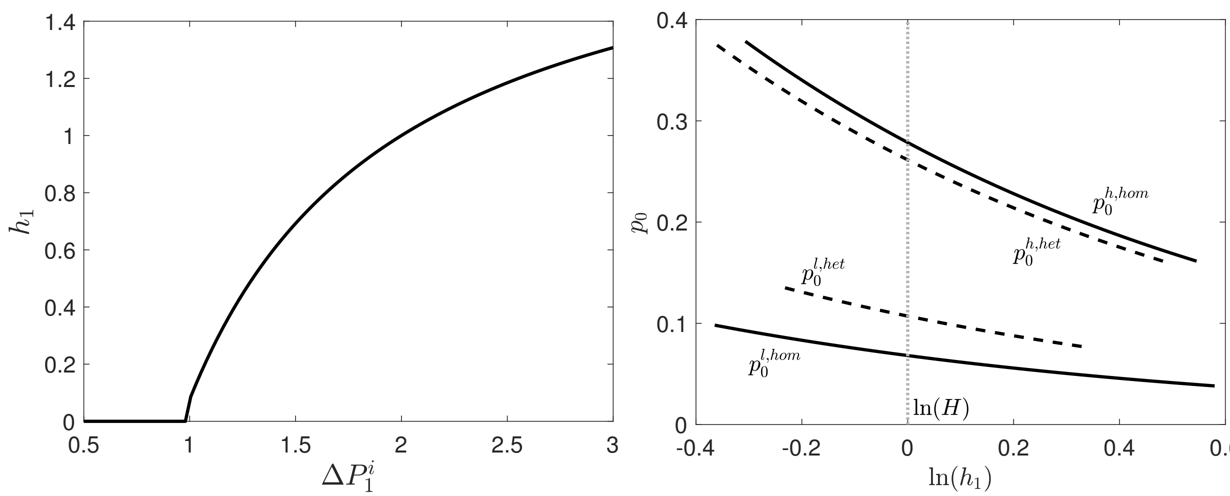
Since the demand function is concave for high house price growth expectations, Jensen's inequality immediately implies that demand with homogeneous expectations will be larger than demand in the corresponding heterogeneous expectations case. This is, for example, the case for $\bar{\Delta}P_1^h = 2$ in Panel (a) of Figure 1, where a mean preserving spread in expectations implies lower demand and therefore lower prices (under the additional assumption that κ is not too large).

This implies equilibrium house prices in the homogeneous and the heterogeneous expectations scenario relate as

$$p_0^{l,hom} < p_0^{l,het}, \qquad p_0^{h,hom} > p_0^{h,het}, \qquad \Rightarrow \qquad p_0^{h,hom} - p_0^{l,hom} > p_0^{h,het} - p_0^{l,het}. \tag{3}$$

Panel (b) displays the associated inverse demand functions in regimes $j \in l, h$ and scenarios $s \in \{hom, het\}$, plotted against log housing, $\ln(h_1)$ and the log housing supply $\ln(H) = 0$. The equilibrium house prices feature exactly the schedule in (3).

Figure 1: Illustration: Housing Demand and Housing Market Equilibrium



(a) Housing Demand in Expectations

(b) Housing Market Equilibrium

Notes: Parametrization: $R = 1, y = 1, a_0 = 0.15, \beta = 0.75, \gamma = 0.1, \bar{\Delta}P_1^l = 1.1, \bar{\Delta}P_1^h = 2, \kappa = 0.5$. Panel (a): Housing demand as a function of house price growth expectations evaluated at $p_0^{h,hom} = 0.28$. Panel (b): Inverse housing demand as function of $ln(h_1)$. Equilibrium house prices: $p_0^{l,hom} = 0.07 < p_0^{l,het} = 0.11, \qquad p_0^{h,hom} = 0.28 > p_0^{h,het} = 0.26$.

2.4 Conclusions for Subsequent Quantitative Analyses

Consider now a sequence of two-period lived households. Then the main take-away from our analysis is that under sufficient movement of house price growth expectations across "regimes" and an according distribution of these expectations, equilibrium house prices may relate as in equation (3). On the basis of these insights the main quantitative questions we pose in our subsequent data analysis in Section 3 as well as in our development and analysis of the structural model in Sections 4 through 7 are, first, whether subjective house price growth expectations elicited in the survey data are in line with these features; second, whether they affect economic decisions in the postulated manner, i.e., when feeding these expectations into a quantitative model of housing demand resulting equilibrium house price movements are in line with (3); third, whether differences between a quantitative model with heterogeneous and homogeneous house price growth expectations are quantitatively relevant; fourth, whether a model with heterogeneous house price growth expectations moves us closer to the data; fifth, as a subsidiary quantitative question, which constraint is the most relevant one to generate concavity in the demand schedule, the DTI constraint—which we looked at in the two-period model—or the loan-to-value constraint—which we will additionally introduce in the quantitative model.

3 House Price Growth Expectations and Housing Decisions

In this section, we explore the panel data on households' house price growth expectations and how those relate to house-adjustment decisions.[6]

The Netherlands experienced a boom-bust-boom cycle over the sample period. Figure 2 shows the time series for annual house price growth, in real terms. There was very fast house price growth in the early 2000s, monotonically declining but positive house price growth until 2008, negative house price growth between 2009 and 2014, and positive house price growth since 2015. House prices increased by about 0.3% per year in real terms on average over the entire period.

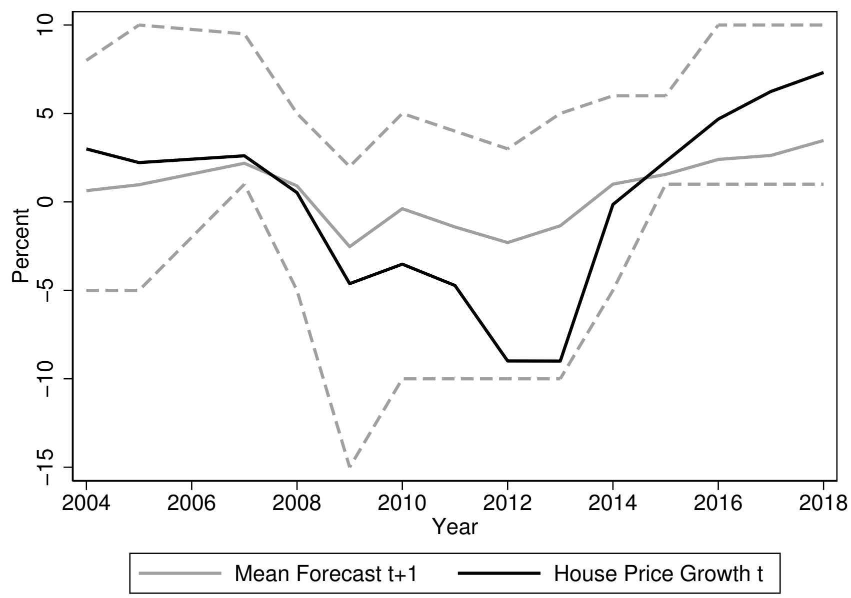
Figure 2 also shows expectations of house price growth by households. The data are households' responses to the two following questions in the Dutch National Bank (DNB) Household Survey: "What kind of price movement do you expect on the housing market in the next two years? Will housing prices increase, decrease or remain about the same?" and "How much percentage points a year will they increase/decrease on average?"[7] Preceding questions in the survey do not reveal whether households are nudged to think about this as a real or a nominal question. Since the average answer over the sample roughly equals the time series average of realized *real* house price growth over this period, we interpret their answers as the answers to a question about real house price growth. Households' average forecast of short-term house price growth is about zero in year 2004, slightly positive in the period 2005-2009, slightly negative in the period 2010-2014, and slightly positive since 2015.

Households' average forecast masks a lot of heterogeneity. The dotted lines in Figure 2 contain 90% of the cross-sectional distribution of house price growth expectations. There is large heterogeneity in house price growth expectations at any given point in time. To further describe

[6]Details on the data and the sources can be found in the Appendix.

[7]We trim the expectations data by dropping the top 1% and the bottom 1% to delete observations with very extreme house price growth expectations.

Figure 2: House Price Growth and House Price Growth Expectations



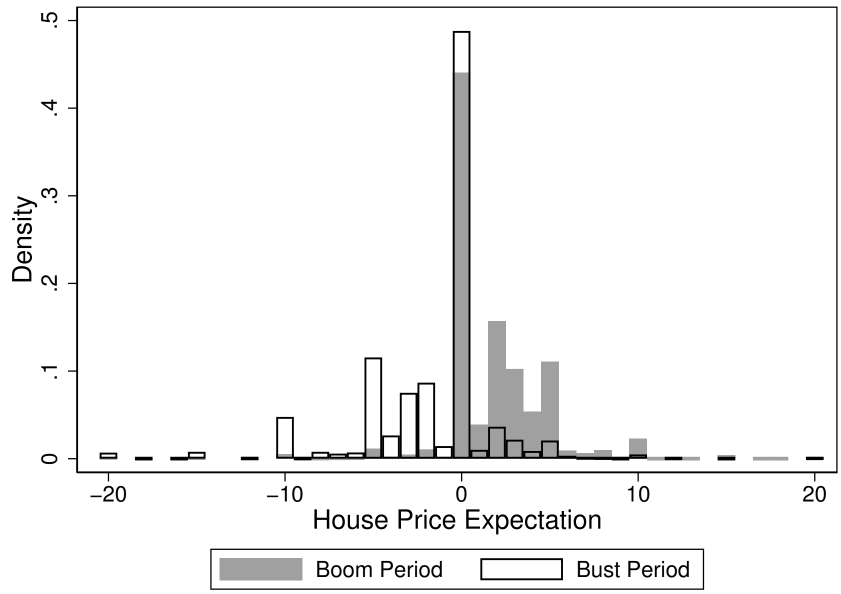
Notes: This figure shows the national house price index net of HIPC inflation in the Netherlands (black solid line) and average expected short-term house price growth for the full sample (gray solid line). Error bands shown in dotted lines contain 90% of the cross-sectional distribution of house price growth expectations.
Source: Own calculations based on DNB Household Survey and ECB Statistical Data Warehouse.

the data on house price growth expectations, we plot in Figure 3 the distribution of the forecast of aggregate house price growth for the two phases of the boom-bust-boom cycle. Specifically, we define as boom periods the years when realized house price growth was positive (years 2004-2008 and years 2015-2018) and as bust period the years in which it was negative (years 2009-2014). The graph shows that there is large heterogeneity in boom periods and in bust periods.

In addition to the question on short-term house price growth expectations, the survey contains a question on long-term house price growth expectations: "In about a period of 10 years what do you think is a normal increase or decrease for property prices **per year**?"

Table 1 reports summary statistics for the short-term house price growth expectations and for the long-term house price growth expectations for the entire sample period (upper panel), the boom periods (middle panel), and the bust period (lower panel). As in Figures 2 and 3, one can see that the average short-term house price growth expectation is much higher in boom periods than in the bust period; whereas the average long-term house price growth expectation is quite similar in the boom periods and in the bust period. In Table 2 we provide results on a regression of house price growth expectations on a constant and a dummy variable for the bust period. In the left column the dependent variable is the short-term house price growth expectation and in the right column the dependent variable is the long-term house price growth expectation. While the

Figure 3: Distribution of Short-Term House Price Growth Expectations



Notes: This figure shows household short-term expected house prices for the full sample, divided into two periods. Boom periods are identified as years with positive house price growth (2004-2008 and 2015-2018); bust periods are years with negative house-price growth (2009-2014).
Source: Own calculations based on DNB Household Survey.

Table 1: Summary Statistics on House Price Growth Expectations

	Mean	St. Dev.
	Entire Sample Period	
Short Term	0.60	3.17
Long Term	2.85	3.44
	Boom Periods	
Short Term	1.71	2.59
Long Term	2.92	3.14
	Bust Period	
Short Term	-1.34	3.15
Long Term	2.77	3.78

Notes: Boom periods are identified as years with positive house price growth (2003-2008 and 2014 to 2018); bust periods are years with negative house-price growth (2009-2013). All variables are measured in percent.
Source: Own calculations based on DNB Household Survey.

dummy variable on the bust period is significant in both regressions (at the 5 percent level), the magnitude of the point estimate is much smaller in the regression for the long-term house price growth expectation. We can therefore conclude that the average short-term house price growth expectation is positive in a boom period and negative in a bust period, whereas the average long-term house price growth expectation is relatively stable over the cycle.[8]

Table 2: Expectations During Booms and Bust

	Short-Term Expectations	Long-Term Expectations
Constant	1.7977***	3.2228***
	(0.0321)	(0.0580)
Bust Period Dummy	-3.3708***	-0.2280**
	(0.0721)	(0.1017)
Observations	16061	10956
R^2	0.1458	0.0005

Notes: Independent variables are short-term expected house-price growth and long-term expected house-price growth; both variables are in percent. Boom periods are identified as years with positive house price growth (2003-2008 and 2014 to 2018); bust periods are years with negative house-price growth (2009-2013). All variables are measured in percent. Robust standard errors in parentheses with $^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$.
Source: Own calculations based on DNB Household Survey.

Likely vs. Unlikely Movers In the data, we also observe the cross-sectional joint distribution of house price growth expectations, income, wealth, and age. To visualize that house price growth expectations are correlated with income, wealth, and age in some years, we perform the following exercise. We first define "likely" and "unlikely" movers in the data based on income, wealth, and age. More specifically, we identify these households on the basis of the predicted moving probability from a linear probability model. The linear probability model regresses a moving indicator variable on income, wealth, and age of the households, as well as year fixed effects.[9] See Table 3. If a household-year observation has a predicted likelihood of moving larger than the average moving rate in the sample of 0.02, they are labeled as "likely movers"; otherwise, they are labeled as "unlikely movers." We then ask whether the households who are likely to move based on income, wealth, and age have different house price growth expectations than the households who are unlikely to move based on income, wealth, and age. Figure 4 plots the average house price growth expectation conditional on belonging to either of the two groups. We find that households who are likely to move based on income, wealth, and demographics hold higher house price growth expectations than households who are unlikely to move based on income, wealth,

[8]The results on the cyclicality of short-term house price growth expectations are robust to excluding focal point answers at 0, which are quite prevalent for short-term house price growth expectations, cf. Figure 3 showing a mode of the distribution at 0. For long-term house price growth expectations, there are no focal point answers at zero and the mode of the distribution is at 2%.

[9]The moving indicator is constructed as a dummy variable from the survey question "WOD35B: In which year did you buy your current house?"

and demographics until about 2012.[10] In the structural housing model in the following section, we will use the full cross-sectional joint distribution of house price growth expectations, income, wealth, and age. The only purpose of Figure 4 is to visualize that house price growth expectations are correlated with the other variables in some years.

Table 3: Moving Propensity Linear Probability Model

	House Adjustment Indicator
Net Financial Assets	0.0000
	(0.0004)
House Value	0.0005
	(0.0005)
Net Income	0.0136***
	(0.0033)
Age	-0.0057***
	(0.0011)
Age squared	0.0000***
	(0.0000)
Renter	0.0165**
	(0.0068)
Constant	0.1769***
	(0.0323)
Year Fixed Effects	Yes
Observations	10352
R^2	0.0200

Notes: Independent variables is an indicator function for house-adjustment. Net income and household portfolio items are in thousands of euros. Renter is a dummy variable for renting households. Robust standard errors in parentheses with $^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$.

Source: Own calculations based on DNB Household Survey.

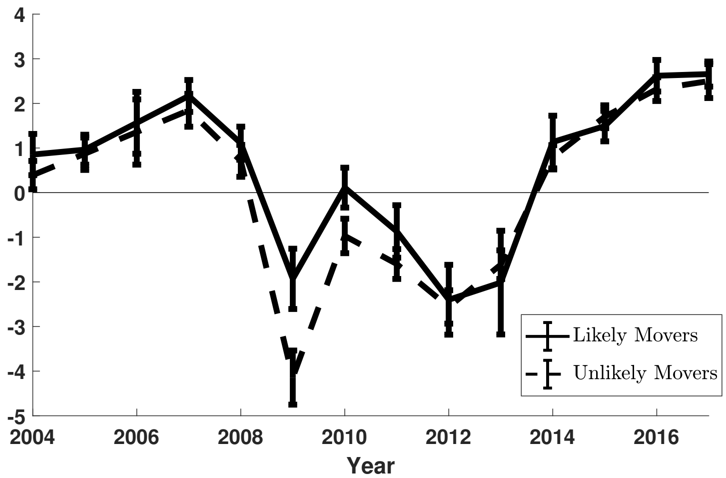
Expectations and Housing Adjustments Based on Quintana (2023) we finally look in Figure 5 at the relationship between house price growth expectations and housing adjustments. Panel (a) focuses on the extensive margin for homeowners: the fraction of adjusting households, conditional on owning.[11] Panel (b) turns to the intensive margin for homeowners: the percentage change in the reported housing value, conditional on owning and moving.[12] We pool across all sample years and group households by their short-term house price growth expectations. The

[10]While the difference is insignificant for most years in our sample period, it is significant in the bust year of 2009 and in the following year.

[11]We use the terms "adjusting" and "moving" interchangeably.

[12]The "reported housing value" is given in the survey as "B26OGB" and is based on the question "WOD44S: In order to calculate for example the deemed home ownership value (eigenwoningforfait) and the immovable property tax (OZB) the government uses the WOZ-value of your house (the official value of your house determined by the municipality). What is the determined WOZ-value for your home?"

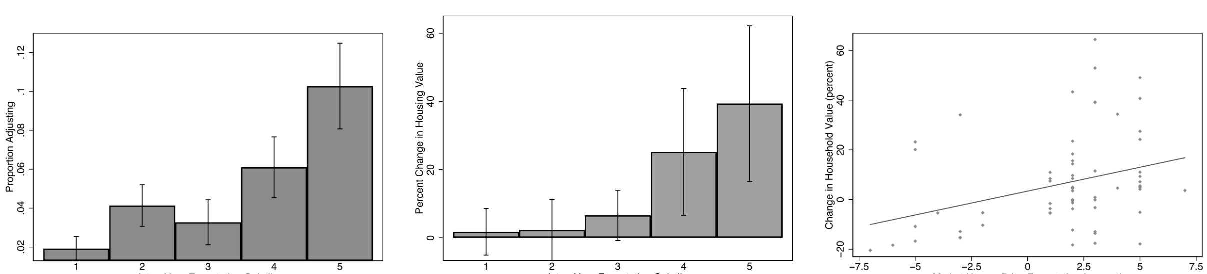
Figure 4: House Price Growth Expectations by Likelihood of Moving, Data



Notes: Likelihood of moving is in percent. Likely movers identified as households with a likelihood of moving higher than 2%. Yearly confidence bands are shown for 95% confidence.
Source: Own calculations based on DNB Household Survey.

fraction of adjusting households, conditional on owning, increases in the short-term house price growth expectation. The size of the housing adjustment, conditional on owning and adjusting, also increases in the short-term house price growth expectation. Specifically, moving homeowners in the lowest expectations quintile on average do not change the value of their home, while moving homeowners in the highest expectations quintile on average increase it by 40%.

Figure 5: Adjusting Households by Expectations Quintiles



(a) Fraction of Adjusting Households　(b) Size of Housing Adjustments　(c) Housing Adjustments in Percent

Notes: This figure shows the fraction of the population adjusting in Panel (a), the percent of housing value adjustments (in 2002 prices) conditional on owning a house in Panel (b) by intra-year short-term expectations quintile, and the percent of house adjustments against expected house-price growth expectations in Panel (c), respectively. Only home-owning households that report well-defined expectations are included in the graphs.
Source: Own calculations based on DNB Household Survey.

Summary of Data Insights The main takeaway from our data exercise is thus threefold. First, households display substantial heterogeneity in their expectations with respect to house price growth. Second, mean short-term house price growth expectations correlate with the observed boom-bust-boom cycle in the Dutch housing market. Third, short-term expectations are also correlated with housing decisions in an expected manner: households that hold higher house price growth expectations are more likely to move and the higher the expected house price growth, the higher are housing adjustments. Our next objective is to develop a quantitative model to investigate how subjective expectations, through the lens of the model, translate into house price dynamics, both in terms of the level as well as changes over time.

4 A Structural Housing Model with Subjective House Price Growth Expectations

In order to study the effects of heterogeneity in house price growth expectations on the level of house prices, we turn to a quantitative structural model. The model features idiosyncratic income shocks, warm glow bequests, home-ownership and rental markets for housing services, and long-term mortgage contracts. We abstract from default as this option is essentially not observed in the Dutch data set.[13] The model is standard and very close to those of Kaplan, Mitman, and Violante (2020) and Berger, Guerrieri, Lorenzoni, and Vavra (2018), but we allow for heterogeneity in house price growth expectations.

We model households in discrete time and denote each period by $t = 0, 1, 2, \ldots$. Our model is cast in partial equilibrium. Interest rates on savings and borrowing are exogenous objects and so are tax instruments, whereas prices for housing and renting units are endogenous.

4.1 Endowments, State and Choice Variables

The model economy is populated by a continuum of households indexed by i. They live with certainty for a fixed number of periods, $j \in \{0, 1, ..., J\}$. During the working period until the fixed retirement age $0 < j_r < J$, households receive a stochastic net labor income with three components: a deterministic and age-specific earnings component $g(j) > 0$, a persistent income state $\eta' = \eta^{\rho}\nu$, where $\rho \in (0, 1)$ is the autocorrelation parameter and $\nu \sim_{i.i.d.} \Psi_{\nu}$ is the current period persistent income shock, and a transitory income shock $\epsilon \sim_{i.i.d.} \Psi_{\epsilon}$. Thus, income during the working period is $y(j; \eta, \epsilon) = g(j)\eta\epsilon$. Retirement income, which in our model encompasses all non-interest old-age income, is proportional to the income received in the period before entering retirement, that is, income for all ages $j \in \{j_r, \ldots, J\}$ is $y(j; \eta_{j_r-1}, \epsilon_{j_r-1}) = \varrho \cdot y(j_r - 1; \eta_{j_r-1}, \epsilon_{j_r-1})$.

[13] As Geis and Luca (2021) observe, "Lenders in the Netherlands have full legal recourse on mortgage borrowers, creating strong incentives to service rather than default on debt. In the wake of the global financial crisis, mortgage NPLs remained comparatively stable, despite a substantial decline in house prices."

Households can save in risk-free bonds that pay a net return, r. Households may also save in discrete housing units, $h' \in \mathcal{H} = \{h_0, ..., h_{n_h}\}, 0 < h_0 < ... < h_{n_h}$, that sell at current period unit price p_t. Since we denote by h the beginning of period housing stock, h' is the housing stock that a household holds during the period and transfers to the next period, forming the beginning of next period's housing stock. When purchasing housing units, households have the option to finance part of the purchase through a loan contract at a fixed rate, r_m, that is subject to an intermediation spread such that $r_m = r + \zeta$, where $\zeta > 0$ denotes the spread. As an alternative to owning—importantly, we do not allow for owning and renting at the same time—, households may choose to live for rent $b \in \mathcal{B} = \{b_0, ..., b_{n_b}\}, 0 < b_0 < ... < b_{n_b}$, where discrete renting units sell at price q_t. It is understood that the elements in $\mathcal{H}$ and $\mathcal{B}$ represent both the size and the quality of houses, respectively apartments, traded in the market.

Housing units depreciate at rate δ and the value of a house owned at the beginning of period t is thus $(1-\delta)p_t h$. If a household decides to adjust the size of the house it owns or decides to change from owning to renting or from renting to owning, it must incur a housing transaction cost linked to the size of the beginning-of-period house, $\theta(1-\delta)p_t h$ for $\theta > 0$. At the beginning of each period, homeowners are also subject to a moving shock, $\xi \in \{0, 1\}$, where the realization $\xi = 1$ occurs with probability $0 < \pi < 1$ and forces a homeowner to sell the house, which changes the financial wealth position at the beginning of period t by $(1-\theta)(1-\delta)p_t h$.[14]

As in Landvoigt (2017), given the positive spread $\zeta > 0$, households will never choose to take out a mortgage and save in bonds at the same time. We therefore only need to keep track of a household's net non-housing (liquid) asset position, which we denote by a. Mortgage contracts are such that, at origination, house *adjusting* and *non-adjusting* households are subject to a maximum debt-to-income (DTI) constraint, $a' \geq -\lambda_y y(j; \eta, \epsilon)$, and a home equity lines of credit (HELOCs) constraint, which we also refer to as a loan-to-value (LTV) borrowing constraint. We follow Kaplan, Mitman, and Violante (2020) by assuming that HELOCs are one-period non-defaultable contracts. Hence, we assume that $a' \geq -\lambda_h p_t h'$. Taking both constraints together we thus have $a' \geq -\min\{\lambda_h p_t h', \lambda_y y(j; \eta, \epsilon)\}$. That is, homeowners are allowed to borrow up to a proportion λ_h of the value of their home, as long as they pay back the loan before they die—there is no possibility to default in the model—, and up to a multiple λ_y of their income. *Renting* households are subject to a zero borrowing constraint $a' \geq 0$. Households begin their economic life with some given housing wealth $h(j = 0) \geq 0$ and some financial assets, $a(j = 0)$.

To summarize, the budget constraint of a household is

$$c + a' + x(d'; h', b, p_t, q_t) = w_t(j; a, h; \eta, \epsilon),$$

[14]After retirement, the moving shock is the only risk that households face. This moving shock generates some renters among the retired households, which is a feature of the data, see Section 6.

where $w_t(j; a, h; \eta, \epsilon)$ is beginning of period total wealth

$$w_t(j; a, h; \eta, \epsilon) \equiv y(j; \eta, \epsilon) + \left(1 + r + \mathbb{1}_{\{a<0\}}\zeta\right) a + (1 - \delta)\, p_t h, \tag{4}$$

and $x(d'; h', b, p_t, q_t)$ are the period t housing expenditures

$$x(d'; h', b, p_t, q_t) = \begin{cases} p_t h' + \theta\,(1-\delta)\, p_t h & \text{if } d' = adj, \text{ i.e., } b = 0, h' > 0, h' \neq h \\ p_t h' & \text{if } d' = nadj, \text{ i.e., } b = 0, h' = h > 0 \\ q_t b + \theta\,(1-\delta)\, p_t h & \text{if } d' = rnt, \text{ i.e., } b > 0, h' = 0. \end{cases}$$

Here $d' = adj$ if a household owns and adjusts during the period, $d' = nadj$ if a household owns and does not adjust during the period, and $d' = rnt$ if a household rents during the period. The borrowing constraint is

$$a' \geq \bar{a} \equiv \begin{cases} -\min\left\{\lambda_h p_t h', \lambda_y y(j; \eta, \epsilon)\right\} & \text{if } \quad h' > 0 \\ 0 & \text{otherwise.} \end{cases}$$

4.2 Preferences

Households derive utility from non-durable consumption, c, and the service flow from housing units owned during the period, h', or from renting an apartment, b. We denote this service flow by $s(h', b, j)$. This service flow $s(\cdot)$ also depends on the age j of the household reflecting that the relative utility of owning versus renting plausibly varies with age. Households discount the future at rate β and the per period utility function $u\left(c, s\left(h', b, j\right)\right)$ satisfies $u_c > 0, u_s > 0, u_{cc} < 0, u_{ss} < 0, u_{cs} = u_{sc} \geq 0$. In the terminal period J, households also value wealth w' they leave behind according to a warm glow bequest utility function $v(w')$ with $v_{w'} > 0$ and $v_{w'w'} < 0$.

4.3 Objective and Subjective Expectations

In each period, households hold *correct* expectations with respect to the persistent income shock, ν, the transitory income shock, ϵ, and the moving shock, ξ. There are no aggregate shocks in the perceived income process and the interest rate r is constant.

Households hold *subjective* expectations with respect to the per period house price growth rate, $\Delta p_{t+s} = \frac{p_{t+s}}{p_{t+s-1}} - 1$, $s \geq 1$. Household i believes in period t that, for sure, the house price growth between t and $t+1$ will equal Δ_i. Hence, household i's period-t expectation of house price growth between t and $t+1$ equals

$$\mathbb{E}_t^i\left[\Delta p_{t+1}\right] = \Delta_i. \tag{5}$$

Furthermore, household i believes in period t that, with probability λ, new information will arrive in period $t+1$ that will make the household expect house price growth of Δ_L in all future periods. Household i believes in period t that, with probability $1-\lambda$, the new information will not arrive in period $t+1$ and her short-term house price growth expectation will remain unchanged in period $t+1$. Hence, household i's period-t expectation of house price growth between $t+1$ and $t+2$ equals

$$\mathbb{E}_t^i\left[\mathbb{E}_{t+1}^i\left[\Delta p_{t+2}\right]\right] = (1-\lambda)\Delta_i + \lambda\Delta_L. \tag{6}$$

Finally, household i believes in period t that the information has a Poisson arrival rate of λ in any period $t+s$, $s \geq 1$. Hence, household i's period-t expectation of house price growth between $t+s$ and $t+s+1$, $s \geq 1$, equals

$$\mathbb{E}_t^i\left[\mathbb{E}_{t+s}^i\left[\Delta p_{t+s+1}\right]\right] = (1-\lambda)^s\Delta_i + \left(1-(1-\lambda)^s\right)\Delta_L, \tag{7}$$

where $(1-\lambda)^s$ is the probability that the information has not arrived by period $t+s$ and $1-(1-\lambda)^s$ is the probability that the information has arrived by period $t+s$. Equations (5)-(7) characterize household i's period-t expectation of the path for the house price growth rate. As an alternative, one could have assumed that household i expects in period t a deterministic decay of the house price growth rate at rate λ to Δ_L. This would have yielded the same equations (5)-(7). We chose to assume expected stochastic decay instead of expected deterministic decay because it yields a particularly transparent formulation of the dynamic programming problem, which we turn to in Section 4.5[15] In the calibration, we set Δ_i equal to the household's short-term house price growth expectation and we set Δ_L equal to the average long-term house price growth expectation.

4.4 The Housing Capital Gains Mechanism

In the presence of movements in the house price, the household must account for potential housing capital gains. Equation (4) implies that next period's beginning-of-period total wealth is

$$\begin{aligned} w' &= y(j+1;\eta',\epsilon') + \left(1+r+\mathbb{1}_{\{a'<0\}}\zeta\right)a' + (1-\delta)\,p_{t+1}h' \\ &= y(j+1;\eta',\epsilon') + \left(1+r+\mathbb{1}_{\{a'<0\}}\zeta\right)a' + (1-\delta)\,p_t h' + \underbrace{(1-\delta)\,p_t\Delta p_{t+1}h'}_{\text{Housing Capital Gains}}. \end{aligned} \tag{8}$$

[15] This modeling choice is similar to the notions of "stochastic aging" or "stochastic retirement" often encountered in the literature.

Hence, conditional on current-period saving and housing choices, the household expectation of its future beginning-of-period resources is:

$$\mathbb{E}_t^i[w'] = \mathbb{E}\left[y(j+1;\eta',\epsilon') \mid \eta\right] + \left(1+r+\mathbb{1}_{\{a'<0\}}\zeta\right)a' + (1-\delta)\,p_t h' + \underbrace{(1-\delta)\,p_t h'\mathbb{E}_t^i\left[\Delta p_{t+1}\right]}_{\text{Expected Housing Capital Gains}}, \quad (9)$$

where $\mathbb{E}_t^i[\Delta p_{t+1}]$ is the period-t expectation of household i of house price growth between t and $t+1$. We allow for heterogeneity in this expectation.

Importantly, whether or not the household receives next-period housing capital gains depends on its current-period housing choice. Hence, one of the mechanism through which subjective expectations about future house price growth affect consumption, savings and housing decisions works through a wealth/endowment effect due to expected future capital gains.

4.5 Dynamic Programming Problems

We describe the dynamic programming problem of a household holding house price growth expectations Δ_L (henceforth, "expectations type $e = L$") followed by households holding house price growth expectations Δ_i (henceforth, "expectations type $e = S$"). Throughout, it is convenient to collect state variables as $z = [\mathbb{E}_t^i[\Delta p_{t+1}], j; a, h, \eta, \epsilon]$. All state variables are summarized in Table 4. Notice that for both expectations types $e \in \{S, L\}$ the terminal value function from the perspective of a period t age j household is

$$V_{t+J-j+1}(z_{t+J-j+1}, e) = v(w'(J))$$

where $w'(J) = \left(1+r+\mathbb{1}_{\{a'(J)<0\}}\zeta\right)a'(J) + (1-\delta)p_{t+J-j+1}h'(J)$.

Table 4: State Variables

State Var.	Values	Interpretation
$\mathbb{E}_t^i[\Delta p_{t+1}]$	$\in \mathbb{R}$	Short-Term House Price Growth Expectation
j	$j \in \{0,\ldots,J\}$	Age of household
t	$t \in \{0,1,2,\ldots\}$	Time
e	$e \in \{S, L\}$	Expectations type
a	$a \geq \bar{a}$	Beginning of period financial assets
h	$h \in \{h_0,\ldots,h_{n_h}\}$	Beginning of period housing wealth
η	$\eta \sim \Psi_\eta$	Persistent income state
ϵ	$\epsilon \sim \Psi_\epsilon$	Transitory income shock

Notes: This table summarizes the state variables of the quantitative model.

Households with House Price Growth Expectations Δ_L. At all ages $j \in \{0, \ldots, J\}$ a household may choose between the three alternatives "owning", "adjusting" and "renting", $d \in \{own, adj, rnt\}$. "Owning" means that the household owns a house at the beginning of the period and attempts to non-adjust the house during the period. The household is then hit by the moving shock realization $\xi = 1$ with probability π. In case the moving shock realizes ($\xi = 1$), the household is forced to sell the house and can purchase a new house or rent. "Adjusting" means that the household adjusts the size of the house or becomes a homeowner during the period. In this case, the moving shock is irrelevant. "Renting" means that the household rents a house during the period. With this notation, we can define the value function as the upper envelope of the choice-d-specific value functions:

$$V_t\left(z, e = L\right) = \max_{d \in \{own, adj, rnt\}} \left\{V_t\left(z, e = L; d\right)\right\},$$

where the choice-d-specific value functions and dynamic problems are

$$\begin{aligned}
V_t\left(z, e = L; d = own\right) &= \pi \max_{d' \in \{adj, rnt\}} \left\{V_t\left(z, e = L; d' = adj\right)\right\} + (1-\pi) V_t\left(z, e = L; d' = nadj\right) \\
V_t\left(z, e = L; d' = adj\right) &= \max_{\{c, a', h'\}} \left\{u\left(c, s\left(h', b = 0, j\right)\right) + \beta \mathbb{E}_t\left[V_{t+1}\left(z', e = L\right)\right]\right\} \\
s.t. \quad c + a' + p_t h' &= y(j; \eta, \epsilon) + \left(1 + r + \mathbb{1}_{\{a<0\}}\zeta\right) a + (1-\theta)(1-\delta) p_t h \\
a' &\geq -min\left\{\lambda_h p_t h', \lambda_y y(j; \eta, \epsilon)\right\} \\
V_t\left(z, e = L; d' = nadj\right) &= \max_{\{c, a'\}} \left\{u\left(c, s\left(h' = h, b = 0, j\right)\right) + \beta \mathbb{E}_t\left[V_{t+1}\left(z', e = L\right)\right]\right\} \\
s.t. \quad c + a' + \delta p_t h' &= y(j; \eta, \epsilon) + \left(1 + r + \mathbb{1}_{\{a<0\}}\zeta\right) a \\
a' &\geq -\min\left\{\lambda_h p_t h', \lambda_y y(j; \eta, \epsilon)\right\} \\
V_t\left(z, e = L; d = rnt\right) &= \max_{\{c, a', b\}} \left\{u\left(c, s\left(h' = 0, b, j\right)\right) + \beta \mathbb{E}_t\left[V_{t+1}\left(z', e = L\right)\right]\right\} \\
s.t. \quad c + a' + q_t b &= y(j; \eta, \epsilon) + \left(1 + r + \mathbb{1}_{\{a<0\}}\zeta\right) a + (1-\theta)(1-\delta) p_t h \\
a' &\geq 0.
\end{aligned}$$

Since households that are forced to move are in the same position as households who move voluntarily, $V_t\left(z, e = L; d' = adj\right) = V_t\left(z, e = L; d = adj\right)$.

Households with House Price Growth Expectations Δ_i. Households with short-term house price growth expectations Δ_i solve almost identical dynamic programming problems. The only difference is that their continuation value is

$$V_{t+1}(z') = \sum_{e' \in \{S,L\}} \pi(e' \mid e = S) V_{t+1}(z', e'),$$

with $\pi(e' = L \mid e = S) = \lambda$ and $\pi(e' = S \mid e = S) = 1 - \lambda$.

4.6 Solution Method

We discretize the income process of the persistent state using the Rouwenhorst method and the transitory shock by Gaussian quadrature, with five and two nodes, respectively. We then apply the method proposed by Sargent, as offered in his online code library, to aggregate and sort the two components into a single ordered income state vector and multiply their respective densities to obtain a single transition matrix for the resulting discrete process. The household model is solved using the discrete-continuous endogenous grid method (DC-EGM) as in Iskhakov, Jørgensen, Rust, and Schjerning (2017). This procedure builds on the EGM of Carroll (2006) and consists of using an exogenous end-of-period (i.e., post-decision) savings grid and the household's Euler equation to back out an endogenous grid for beginning-of-period net financial assets. Secondary kinks in choice-specific value functions are handled by eliminating segments that fall below the upper envelope of the correspondence.

5 Temporary Equilibria and Price Dynamics

In order to study the implications of heterogeneity in house price growth expectations for house prices, we use the structural housing model and look at a sequence of temporary equilibria—in the spirit of Hicks (1939) and Grandmont (1977, 1988)—generated by the empirical distribution of expectations, income, wealth, and demographics. Following Piazzesi and Schneider (2016, p.1587), a temporary equilibrium for date t, is defined as "a collection of prices and allocations such that markets clear given beliefs and agents' preferences and endowments."[16] Further, again following Piazzesi and Schneider (2016, p.1589), "a sequence of temporary equilibria"—again, in our context, with measured expectations—"is a collection of date t temporary equilibria that are connected via the updating of endowments." In our setup, we update endowments by feeding into the model in each period t the joint distribution of income, wealth and expectations from the data.

By modeling the dynamics of house prices as a sequence of temporary equilibria with measured expectations, we account for the effects of distributional changes—including changes in expectations—within the household sector on house prices, while remaining agnostic about the

[16] Also see Farhi and Werning (2019) and Molavi (2019) for recent examples using the concept as well as the review article by Brunnermeier et al. (2021), which discusses the usefulness of the concept in studies with survey beliefs.

source of such changes. In particular, we are agnostic about any specific expectation-formation process that is behind the observed joint distribution of expectations, income, wealth, and demographics. Further, by taking the supply of assets—i.e., the aggregate stocks of financial assets and housing wealth—directly from the data, we do not need to explicitly model the supply side of the economy.[17] In this way, the sequence of temporary equilibria generated by the model allows us to map the *observed* sequence of distributions over expectations and states, $\{\Phi_t\}_{t=2004}^{t=2017}$, to a sequence of price vectors, $\{[p_t, q_t]\}_{t=2004}^{t=2017}$, which includes the boom-bust-boom cycle in the housing market in the Netherlands.

5.1 A Sequence of Temporary Equilibria with Measured Expectations

This section provides a formal definition of a date t temporary equilibrium (with measured expectations) and the sequence of temporary equilibria.

Let $\mathcal{G} = \mathbb{R}$ be the set of all possible house price growth expectations, $\mathcal{J}$ be the set of possible ages, $\mathcal{A} = \mathbb{R}$ be the set of possible non-housing assets held by the household, $\mathcal{H}$ be the set of possible housing assets, $\mathcal{N}$ be the set of possible persistent income state realizations, and $\mathcal{E}$ be the set of possible transitory income shock realizations. Let $z = [\mathbb{E}_t^i[\Delta p_{t+1}], j; a, h, \eta, \epsilon]$ and $\mathcal{Z} = \mathcal{G} \times \mathcal{J} \times \mathcal{A} \times \mathcal{H} \times \mathcal{N} \times \mathcal{E}$. Further, let $\mathcal{P}(\iota)$ and $\mathcal{B}(\iota)$ denote the power set and the Borel σ-algebra of ι, respectively. Finally, let $\mathcal{M}$ be the set of all probability measures on the measurable space $(\mathcal{Z}, \mathcal{B}(\mathcal{Z}))$, where $\mathcal{B}(\mathcal{Z}) = \mathcal{B}(\mathcal{G}) \times \mathcal{P}(\mathcal{J}) \times \mathcal{B}(\mathcal{A}) \times \mathcal{P}(\mathcal{H}) \times \mathcal{B}(\mathcal{N}) \times \mathcal{B}(\mathcal{E})$.

Definition 1 (Temporary Equilibrium)**.** *Given the interest rate r, the loan spread ζ, the supply of owner-occupied housing H_t, the supply of rental housing B_t, and a cross-sectional measure $\Phi_t(z)$, a period t temporary equilibrium is a set of functions $V_t : \mathcal{Z} \to \mathbb{R}$, $c_t : \mathcal{Z} \to \mathbb{R}_+$, $a'_t : \mathcal{Z} \to \mathcal{A}$, $h'_t : \mathcal{Z} \to \mathcal{H}$, and $b_t : \mathcal{Z} \to \mathbb{R}_+^0$, as well as prices $[p_t, q_t]$ such that*

1. *The functions V_t, c_t, a'_t, h'_t, and b_t are measurable with respect to $\mathcal{B}(\mathcal{Z})$, the function V_t satisfies the households' Bellman equation and the functions c_t, a'_t, h'_t, and b_t are the associated policy functions.*

2. *Markets clear*

$$H_t = \int h'_t(z) d\Phi_t(z), \qquad B_t = \int b_t(z)\, d\Phi_t(z), \qquad A_t = \int a'_t(z) d\Phi_t(z). \tag{10}$$

The concept of a period t temporary equilibrium is a generalization of the concept of a rational expectations equilibrium. A period t temporary equilibrium gives the allocations and prices for

[17] Since our results arise from an exogenous sequence of joint distributions, they continue to hold for any model that delivers an identical sequence of equilibrium distributions—regardless of the source of fluctuations and supply-side dynamics. Finally, note that we do not need to treat the distribution of households as a state variable in the household's dynamic programming problem since this would only be relevant—in the presence of aggregate risk—if it informed household's price expectations, which we already directly observe.

any given beliefs, a special case are the beliefs that are given by some model of belief formation (e.g., full-information rational expectations).

A sequence of temporary equilibria is next defined as follows:

Definition 2 (Sequence of Temporary Equilibria)**.** *A sequence of temporary equilibria is a collection of date t temporary equilibria with a sequence of cross-sectional distributions, $\Phi_t(z)$.*

We take the sequence of $\Phi_t(z)$ from the data and remain agnostic about how the sequence of short-term house price growth expectations have been formed. For income and wealth, we thus overwrite in each period t the model generated distribution with the actual distribution as measured from the data. While the implicit income shocks are consistent with the stochastic process we estimated, the implicit shocks to wealth have zero ex-ante probability.

5.2 Computational Implementation

To compute a date t temporary equilibrium, we feed into the model from the data the cross-sectional joint distribution of short-term house price growth expectations, income, financial wealth, owned housing value, and age. We solve the household model as described in Subsection 4.6. For given value and policy functions and a given cross-sectional distribution, we compute household demand by multivariate linear interpolation[18] and solve the market clearing on the housing and rental market, cf. equation (10), as a bivariate rootfinding problem in $[p_t, q_t]$. Given the high degrees of non-linearity, we use the algorithm of Zhang, Conn, and Scheinberg (2010).[19]

6 Functional Forms and Calibration

In this section, we specify the functional forms relating to households' preferences and discuss calibration.

6.1 Functional Forms

Households' instantaneous utility function, following Landvoigt (2017) and Berger, Guerrieri, Lorenzoni, and Vavra (2018), is given by

$$u\left(c, s\left(h', b, j\right)\right) = \frac{\left[c^{1-\sigma} s\left(h', b, j\right)^{\sigma}\right]^{1-\gamma} - 1}{1-\gamma},$$

[18] To interpolate along the income dimension, we deduct from observed income the deterministic income component (predicted from a first stage income regression, cf. Section 6.2), and interpolate the residual stochastic component on the single income state, cf. our description in Section 4.6.

[19] We would like to thank the authors for providing us with the Fortran code that implements their algorithm.

where the service flow of utility from owned houses, respectively from rented apartments, $s(\cdot)$, is linear in its first two arguments and given by

$$s(h', b, j) = \omega_j h' + b + \varpi, \quad \text{where } \varpi \geq 0 \text{ and } \omega_j = 1 + e^{\omega_0 + \omega_1 j + \omega_2 j^2} \geq 1.$$

In the above, parameter ϖ measures the value of social housing as in, e.g., Kaas, Kocharkov, Preugschat, and Siassi (2021), and age dependency of the relative weight parameters ω_j is assumed to match the hump-shaped home ownership profile in the data, see below.

Our specification of the utility from bequests follows De Nardi (2004) and is given by

$$v(w) = \vartheta_1 \frac{(w + \vartheta_2)^{1-\gamma} - 1}{1 - \gamma},$$

where parameter $\vartheta_1 > 0$ measures the level utility derived from intended bequests and parameter $\vartheta_2 \geq 0$ controls the "luxury goods" motive.

6.2 Calibration

We pursue a standard calibration strategy distinguishing between parameters measured from the data (first-stage parameters) and those that are identified using the model (second-stage parameters).

Values of first-stage parameters are reported in Table 5. The model is specified at an annual frequency, with households starting their working life at age 25 ($j = 0$), retiring at age 65 ($j = 40$) and dying at age 80 ($j = 55$). There is no stochastic death between periods. The share of housing services is set to $\sigma = 0.3$ to match the empirical average rental expenditure of 30% of income. The risk aversion parameter is fixed at $\gamma = 2$, in line with much of the macroeconomic literature. The estimation of the income process follows Storesletten, Telmer, and Yaron (2004). We estimate an autocorrelation of the persistent income component of $\rho = 0.97$, and a variance of the persistent shock of $\sigma_\nu^2 = 0.008$ and of the transitory shock of $\sigma_\epsilon^2 = 0.084$. In our model, pension income is all non-interest income households receive in retirement and not just pension income. We therefore focus on the ratio of average old age to working age income in the data, and accordingly set the old age income replacement rate to 0.85. The risk-free rate is set to $r = 0.03$. The mortgage loan markup is set to the period average of $\zeta = 0.01$ p.a.. The maximum DTI and LTV ratios are set to the estimated averages in the data of $\lambda_y = 5$ and $\lambda_h = 0.9$, respectively. We follow Kaplan, Mitman, and Violante (2020) by fixing log-linear housing grids with four and six points for rental and owner-occupied housing, respectively. The minimum and maximum owner-occupied housing grid points correspond to the 10^{th} and 80^{th} percentile of the empirical distribution of houses. The minimum rental grid point is set equal to the 10^{th} percentile of the rental housing distribution and the remaining three grid points are identical to the first three grid points of the owner-occupied housing grid. Specifically,

$$\mathcal{H} = \{0.243, 0.373, 0.571, 0.874, 1.338, 2.048\}$$
$$\mathcal{B} = \{0.098, 0.243, 0.373, 0.571\}.$$

These variables are measured in terms of the median real net annual income. For example, the annual rent at the 10^{th} percentile of the rental housing distribution is $\$3,903$ and the annual income is $\$39,829$, both at 2023 prices, which yields $\frac{3,903}{39,829} = 0.098$. The house price at the 10^{th} percentile of the owner-occupied housing distribution is $\$138,264$. With a rent-to-price ratio of $q/p = 0.07$, see below, this yields $\frac{138,264}{1/0.07 \cdot 39,829} = 0.243$. Following Fernandez-Villaverde and Krueger (2011), the value of social housing is treated like a computational parameter and set to a level low enough so that it does not have a noticeable impact on the implications of the model, $\varpi = 0.00001$. The house depreciation rate, $\delta = 0.04$, is a plausible parameter value for the Netherlands, and the house sales transaction cost, $\theta = 0.07$, is taken from Kaplan, Mitman, and Violante (2020).

Table 5: First-Stage Parameters

Parameter	Interpretation	Value
	Demographics	
j	Period length in years	1
J	Length of life 80	55
j_r	Retirement age 65	40
	Preferences	
σ	Weight on housing services	0.3
γ	Risk aversion	2
	Expectations	
$\mathbb{E}_t^i[\Delta p_{t+1}]$	Short-term house price growth expectations	Data
Δ_L	Long-term house price growth expectations	Data
$\pi(e' = L \mid e = S)$	Expectations transition probability	0.1
	Income Process	
$\{g(j)\}$	Deterministic age profile polynomial	P(4)
ϱ	Replacement rate	0.85
ρ	Autocorrelation of persistent component	0.97
σ_ν^2	Variance of persistent shock	0.008
σ_ϵ^2	Variance of transitory shock	0.084
	Housing Sector	
p/q	Owner-occupied housing price	$1/0.07$
n_h	No. owner-occupied house sizes	6
n_b	No. rental house sizes	4
ϖ	Value of social housing	0.00001
δ	House depreciation rate	0.04
θ	House sales transaction cost	0.07
	Financial Instruments	
r	Risk-free rate	0.03
ς	Mortgage loan markup	0.01
λ_y	Maximum DTI ratio on mortgage loans	5
λ_h	Maximum LTV ratio on mortgage loans	0.90

Note: This table lists the parameters calibrated using only the data, as well as their economic interpretation and their value.
Source: Own calculations based on DNB Household Survey.

To determine the second-stage parameters, we fix prices and expectations and solve for the stationary distribution of the model. We iterate on the values of the second-stage parameters to match associated data moments. Specifically, we normalize the rental rate to $q = 1$ and set the rent-to-price ratio to $q/p = 0.07$ so that the owner occupied housing price in the model is $p = 1/0.07$, as in Nijskens and Lohuis (2019). Since our model does not feature aggregate shocks, house prices are constant and house price growth expectations are equal to zero at this stage.

Table 6: Second-Stage Parameters

Parameter	Interpretation	Targeted Moment	Value
β	Discount factor	Average net worth	0.965
π	Moving shock hazard rate	Annual percent of moving homeowners	0.018
ϑ_1	Strength of bequest motive	Median $NW_{j=80}$ / Median $NW_{j=50}$	1915.104
ϑ_2	Luxuriousness of bequests	Share of age 80 bequ. HH in bottom half of NW distr.	30.945
ω_0	Additional utility from owning	Homeownership rate	-0.927
ω_1	Additional utility from owning	Polynomial coefficient 1	-0.030
ω_2	Additional utility from owning	Polynomial coefficient 2	-0.00005

Note: This table lists the parameters calibrated using the model, as well as their economic interpretation, the empirical concept which they target, and their value.

Values of second-stage parameters determined this way are reported in Table 6. While all parameters are identified jointly, for expositional purposes we associate each parameter with a specific calibration target. The discount factor is chosen so as to match the average level of net worth of households, giving $\beta = 0.965$. The hazard rate of the moving shock is set to target the average moving rate of homeowners, yielding $\pi = 0.018$.[20] The parameters governing the utility premium due to homeownership $\{\omega_j\}_{j=0}^{2}$ are chosen to match the model-implied age polynomial of homeownership rates with its empirical counterpart after controlling for time and cohort effects. We follow Kaplan, Mitman, and Violante (2020) in setting the parameters of the warm glow bequest motive. The parameter governing the strength of bequests, ϑ_1, is set targeting the median net worth ratio of 50-80 year-old households. Even though, our survey does not include direct evidence on bequests, households are asked whether they intend to bequeath. While in the top half of the net wealth distribution nearly everyone intends to bequeath, in the bottom half only 35.4% intend to do so. Therefore, we calibrate the luxuriousness of bequests parameter, ϑ_2, to match this fraction.

Targeted moments and corresponding model moments are reported in Table 7. Overall, the fit of our model to targeted moments is good. Notice, however, that we face a tension between matching the homeownership rate in old age—and thus net wealth of households—and the percent of households intending to bequeath in the bottom half of the net wealth distribution. Our model undershoots the homeownership rate at age 80, cf. Panel (a) of Figure 6, but overshoots

[20]Since our model abstracts from moving costs for renters, we do not target the average moving rate of renters.

the fraction of households intending to bequeath wealth, cf. Table 7. We could match the latter fraction better by increasing the luxury bequest parameter ϑ_2, but this would deteriorate further the fit to the homeownership rate and to net wealth. For this reason, we do not achieve exact identification, undershoot net wealth and overshoot the fraction of households intending to bequeath.

In our main model experiments in Section 7, we "discretize" short-term house price growth expectations on a grid with seven gridpoints.[21] To calculate the model-implied housing demand of a household with given short-term house price growth expectations $\mathbb{E}_t^i\left[\Delta p_{t+1}\right]$ from the survey, we linearly interpolate between gridpoints. The long-term house price growth expectations are instead restricted to be homogeneous and are set equal to 2%, because 2% is the mode (and the median) of the empirical distribution of long-term house price growth expectation. Finally, we set $\lambda = 0.1$.

Table 7: Calibration Targets and Model Moments

Targeted Moments	Data	Model
Average net worth	7.254	6.722
Homeownership rate	0.731	0.782
Polynomial coefficient 1	0.031	0.067
Polynomial coefficient 2	-0.0002	-0.0006
Median $NW_{j=80}$ / Median $NW_{j=50}$	1.391	1.320
Share of age 80 bequ. HH in bottom half of NW distr.	0.354	0.580
Annual percent of moving homeowners	0.019	0.018

Note: This table lists the moments targeted in calibration, as well as the values in the data and values implied by the model. Household assets are expressed in terms of median annual income.
Source: Own calculations based on DNB Household Survey.

6.3 Life-Cycle Profiles

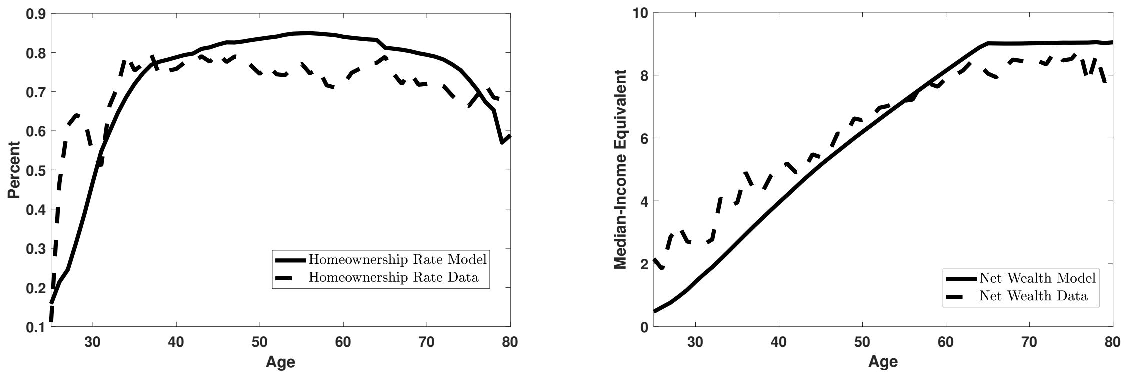
The life-cycle profiles for the baseline calibration are presented in Figure 6. The takeaway is that the model does a decent job in matching the life-cycle profiles of the homeownership rate and of net wealth.

6.4 Adjustment Behavior by Expectation Quintile

It is also useful to compare the model's performance in terms of house adjustment behavior. Figure 5 reports the fraction of adjusting homeowners by expectation quintile and the size of house adjustments, conditional on owning and adjusting, by expectation quintile. Figure 7 shows the same statistics for the model: the fraction of adjusting homeowners by expectation quintile (left panel) and the size of house adjustments, conditional on owning and adjusting, by expectation quintile (right panel). These statistics were not directly targeted in the calibration. Nonetheless,

[21] The values of the grid are $[-0.05, -0.03, 0.0, 0.02, 0.03, 0.05, 0.10]$.

Figure 6: Life-Cycle Profiles

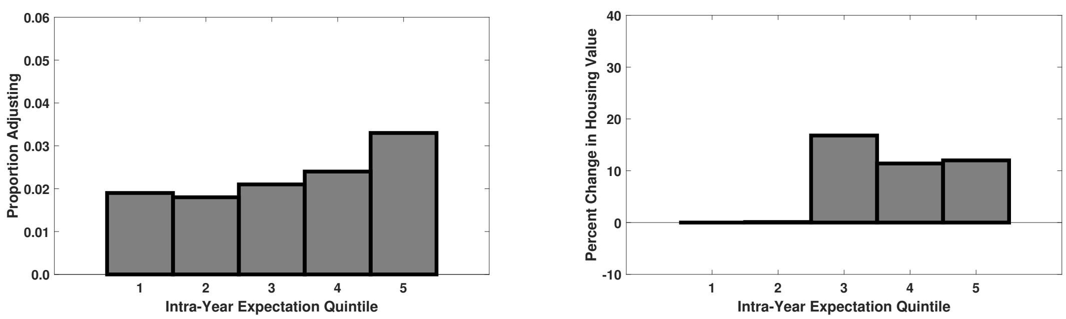


(a) Homeownership Rate (b) Net Wealth

Notes: This figure shows the model-implied and empirical age profiles for homeownership, and net wealth.
Source: Own calculations based on DNB Household Survey.

the model does a fair job in replicating the main patterns identified in the data and is within an acceptable level of accuracy from a quantitative perspective.

Figure 7: Adjusting Households by Expectations Quintiles



(a) Fraction of Adjusting Households (b) Size of Housing Adjustments

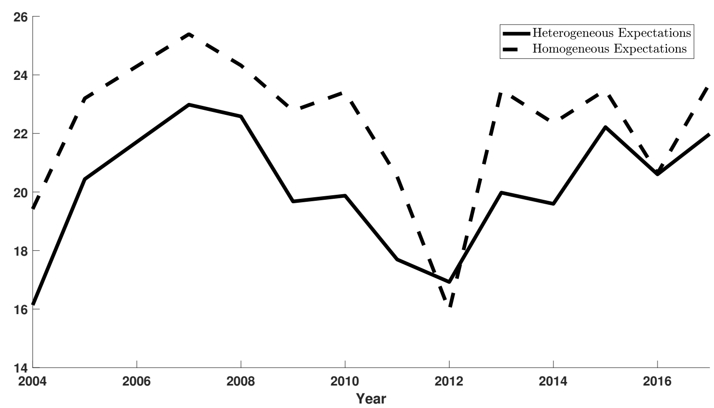
Notes: This figure shows the model-implied fraction of the home-owning population adjusting in Panel (a) and the size (in percent) of housing value adjustments conditional on owning a house in Panel (b) by short-term expectations quintile.
Source: Own calculations based on DNB Household Survey.

7 The Role of Heterogeneous House Price Growth Expectations for Equilibrium House Prices

Our main objective is to investigate the effects of heterogeneity in house price growth expectations on the level and the dynamics of house prices. For this purpose, we report the equilibrium house price sequence for two specifications: (i) the baseline specification with heterogeneous house price growth expectations, as described in the previous section, and (ii) an alternative specification where all households have the same short-term house price growth expectation equal to the cross-sectional average from the baseline specification. By construction, the cross-sectional average of households' short-term house price growth expectations is the same in the two model variants at each point in time. We refer to the first model variant as "heterogeneous expectations" and to the second model variant as "homogeneous expectations".

Figure 8 shows the sequence of equilibrium house prices in the two model variants. We make two important observations. First, both model variants generate a house price boom until 2007, followed by a bust lasting from 2008 until 2012, which is succeeded by another house price boom. This pattern is broadly consistent with the data, cf. Figure 2.[22] Turning to our main question, the model with heterogeneous house price growth expectations features a lower level of the house price, apart from in the trough of 2012, and a lower amplitude of house price fluctuations.

Figure 8: Owner-Occupied House Prices



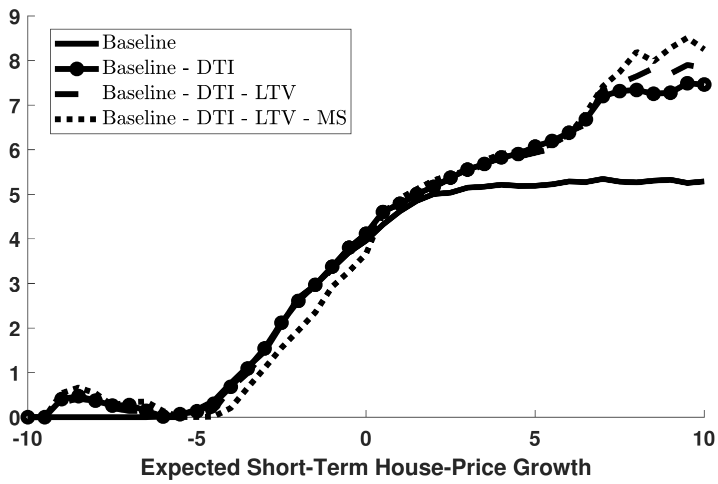
Notes: This figure shows the times series of the model-implied house price under heterogeneous expectations and homogeneous expectations.
Source: Own calculations based on DNB Household Survey.

These findings on how the level and the dynamics of house prices are affected by heterogeneity in expectations may have many reasons. For example, it could be that households with high house price growth expectations tend to be households who are far above the buying threshold, while households with low house price growth expectations tend to be households who are marginally below the buying threshold. In this case, reducing the expectations of the first type of household and increasing the expectations of the second type of household would increase housing demand. We next demonstrate that the main reason for these findings on the level and the dynamics of house prices is the convex-concave shape of housing demand in house price growth expectations.

[22] In the data, the house price growth rate was still marginally positive in 2008 and still negative in 2013-2014.

We first show that housing demand is a convex-concave function of the house price growth expectation. For this purpose, we compute the average demand for houses at different values of the short-term house price growth expectation by aggregating over all other state variables in the model.

Figure 9: Concavity of Housing Demand in Expectations

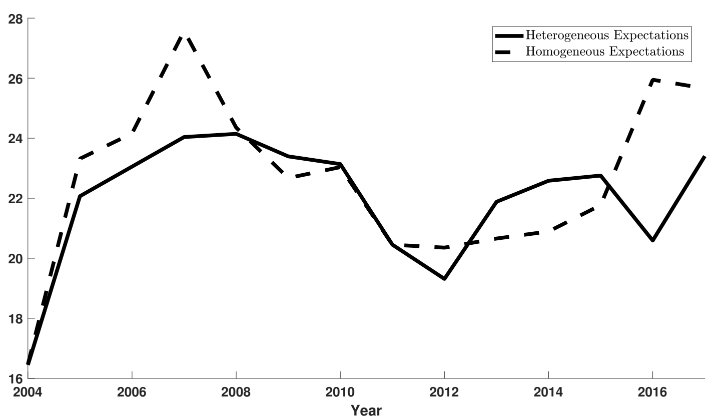


Notes: This figure shows the housing demand (in median income units) for homeowners by short-term expected house price growth, in percent. To compute housing demand we solve a pure household model as in the calibration and aggregate with the according cross-sectional distribution when households hold homogeneous short-term house price growth expectations of $\Delta \in [-10, 10]\%$ and long-term house price growth expectations of 2%.
Source: Own calculations based on DNB Household Survey.

We thereby construct the quantitative model analogue to the homogeneous expectations scenario of Section 2. To do so, we endow households with the same short-term house price growth expectation of Δ, which we vary between -10 and $+10$ percent, give them long-term house price growth expectations of 2%, solve a pure household model as in the calibration and aggregate with the cross-sectional distribution characterized at calibration. Results of this exercise are shown in Figure 9, solid line. We observe that, at a house price growth expectation below -4%, demand for housing in the model is basically zero. At a house price growth expectation above -4%, the average housing demand increases in the house price growth expectation, and beyond the convexity of housing demand in the expectation for values of the house price growth expectation around -4%, housing demand becomes a concave function in the house price growth expectation. Going from the heterogeneous expectations specification to the homogeneous expectations specification, households with high and low expectations might both experience a significant change in their expectations as a result of the homogenization, but the effect of this homogenization on their demand for housing will be asymmetric. As in our illustrative model of Section 2, if house price growth expectations are high on average, then homogenization will increase housing demand and thus equilibrium house prices. In contrast, if house price growth expectations are low on average, then homogenization will reduce housing demand thereby pushing equilibrium house prices down.

To understand the source of the concavity, we next turn off various features of the model; the debt-to-income constraint (DTI), the loan-to-value constraint (LTV), and the moving shock (MS). Even with all three model elements switched off, the housing demand stays concave. See Figure 9, dotted line. However, switching off the DTI constraint, leads to a strong reduction in the concavity. Economically, households with very high house price growth expectations would like to buy very valuable houses on the market to the point where the DTI constraint becomes binding, which suppresses demand relative to a model in which the DTI constraint does not apply.

Figure 10: House Prices without Debt-to-Income Constraint



Notes: This figure shows the times series of the model-implied house price under heterogeneous expectations and homogeneous expectations, whilst shutting off the debt-to-income restriction in both versions of the model. All expectations and state variables correspond to the data.
Source: Own calculations based on DNB Household Survey.

To confirm the importance of the DTI constraint for the effects of heterogeneity in house price growth expectations, we compute the same sequences of temporary equilibria as in Figure 8 but with the DTI constraint switched off. Results shown in Figure 10 confirm the relevance of the concavity of demand in house price growth expectations driven by the DTI constraint. Without it, the level of house prices in the heterogeneous expectations model variant and in the homogeneous expectations model variant are essentially the same, apart from in the boom years around 2007 and in the boom years around 2016. The fact that the equilibrium house price is still higher in the homogeneous expectations model variant than in the heterogeneous expectations model variant in the boom years around 2007 and 2016 is due to the fact that, even without the DTI constraint, housing demand is somewhat concave for positive house price growth expectations and strongly concave at very high house price growth expectations, cf. Figure 9. Hence, the homogenization of house price growth expectations still drives up house prices in times of very high average house price growth expectations.

We also investigate how close the model comes to matching the boom-bust-boom cycle shown in Figure 2. We compute the percentage change in house prices from the peak to the trough of the cycle in the data and in the two model variants, see Table 8. Abstracting from the heterogeneity raises the amplitude from peak to through by 11 percentage points (41 percent). It also raises the level of house prices by 11 percent, see Figure 8. As Table 8 further shows, the heterogeneous expectations specification brings us close to the amplitude of house prices as measured in the data. In contrast, the amplitude in the homogeneous expectations specification is substantially higher than in the data.

Table 8: Amplitude of House Prices in Data and Model

	Data	Heterogeneous Expectations	Homogeneous Expectations
Percent Change	-27.49	-26.35	-37.18

Notes: The amplitude is measured as the percent distance between peak at the beginning of the sample period and trough of the cycle using centralized (model) data. We exclude the first year, year 2004, where house prices in the heterogeneous expectations model are lowest. In both variants of the model, the peak year is 2007 with a centralized real house price index of 1.139 in the heterogeneous and of 1.259 in the homogeneous agent model. The trough year is 2012, where the centralized price indices are 0.839 and 0.790, respectively. In the data, the peak is reached one year later, in 2008, with a centralized index value of 1.151, and the trough two years later than in the model, in 2014, with a value of 0.834.

Source: Own calculations based on DNB Household Survey.

8 Conclusion

House price growth expectations are very heterogeneous in the population. In this paper, we study the effect of heterogeneity in house price growth expectations on the dynamics of house prices and the level of house prices. We solve a structural housing model. In every period, we compute the equilibrium house price for the empirical joint distribution of house price growth expectations, income, wealth, and age. As a counterfactual, we give each household at every point in time the cross-sectional average house price growth expectation at that point in time. We find that abstracting from the empirical heterogeneity in house price growth expectations increases the amplitude of house price fluctuations and the average level of house prices. Put differently, heterogeneity in house price growth expectations reduces the amplitude of house price fluctuations and the average level of house prices. The reason for the lower amplitude under heterogeneous expectations is that, in boom periods, a large fraction of households in the upper part of the cross-sectional distribution of expectations have a binding debt-to-income (DTI) constraint, and giving every household the average expectation does not reduce their housing demand but drives up the housing demand of the other households. In bust periods, a large share of households in the lower part of the cross-sectional distribution of expectations are renting households who are far away from the buying threshold, and allocating to every household the average expectation does not increase their housing demand but drives down the housing demand of the other households. The reason for the lower average level of house prices under heterogeneous expectations is that our sample period is dominated by boom periods, and hence, the first effect dominates on average over the sample period. The loan-to-value constraint does not drive these findings.

Our findings on the role of the DTI constraint suggest that the interaction between subjective house price growth expectations and institutional features of the housing market plays an important role in the determination of house prices.

In future work, it could be interesting to explore the policy implications of the heterogeneity in house price growth expectations. It is feasible to compute policy counterfactuals with the model in this paper, but one would need to add additional assumptions. To fix ideas, suppose one is interested in computing the impulse response of house prices to an announced change in the DTI constraint. First, one needs to add an assumption about how households' expectations of future house prices respond to this announcement. One approach would be to elicit how expectations respond to the announcement using the method in Roth, Wiederholt, and Wohlfart (2023). With the new joint distribution of age, income, wealth, and expectations, and the new policy function that is obtained from the dynamic programming problem with the new DTI constraint, one can compute the new market clearing house price on impact of the announcement. Second, to compute the market clearing house price in the following periods after the announcement period, one needs to add an assumption about how households revise their beliefs about future house price growth based on observed, past, realized house price growth. Here one could estimate a recursive belief updating rule (e.g., of the type implied by a Kalman filtering problem) on the survey data used in this paper. With the model and these two additional assumptions, one can compute the impulse response of house prices to the change in the policy parameter. We leave this to future research.

References

Armona, L., A. Fuster, and B. Zafar (2019). Home price expectations and behaviour: Evidence from a randomized information experiment. *The Review of Economic Studies 86*(4), 1371–1410.

Bailey, M., R. Cao, T. Kuchler, and J. Stroebel (2018). The economic effects of social networks: Evidence from the housing market. *Journal of Political Economy 126*(6), 2224–2276.

Bailey, M., E. Dávila, T. Kuchler, and J. Stroebel (2019). House price beliefs and mortgage leverage choice. *The Review of Economic Studies 86*(6), 2403–2452.

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra (2018). House prices and consumer spending. *The Review of Economic Studies 85*(3), 1502–1542.

Bottan, N. and R. Perez-Truglia (2024). Betting on the house: Subjective expectations and market choices. *forthcoming in: American Economic Journal: Applied Economics*.

Broer, T., A. Kohlhas, K. Mitman, and K. Schlafmann (2022). Expectation and wealth heterogeneity in the macroeconomy. *Mimeo*.

Brunnermeier, M., E. Farhi, R. S. Koijen, A. Krishnamurthy, S. C. Ludvigson, H. Lustig, S. Nagel, and M. Piazzesi (2021). Review article: Perspectives on the future of asset pricing. *Review of Financial Studies 34*(4), 2126–2160.

Burnside, C., M. Eichenbaum, and S. Rebelo (2016). Understanding booms and busts in housing markets. *Journal of Political Economy 124*(4), 1088–1147.

Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters 91*(3), 312–320.

De Nardi, M. (2004). Wealth inequality and intergenerational links. *The Review of Economic Studies 71*(3), 743–768.

Farhi, E. and I. Werning (2019). Monetary policy, bounded rationality, and incomplete markets. *American Economic Review 109*(11), 3887–3928.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh (2017). The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy 125*(1), 140–223.

Fernandez-Villaverde, J. and D. Krueger (2011). Consumption and saving over the life cycle: How important are consumer durables? *Macroeconomic Dynamics 15*(5), 725–770.

Fuster, A., R. Perez-Truglia, M. Wiederholt, and B. Zafar (2022). Expectations with endogenous information acquisition: An experimental investigation. *Review of Economics and Statistics 104*(5), 1059–1078.

Geis, A. and O. Luca (2021). Real estate in the Netherlands: A taxonomy of risks and policy challenges. *IMF Working Paper 21/206*.

Grandmont, J. M. (1977). Temporary general equilibrium theory. *Econometrica 45*(3), 535–572.

Grandmont, J.-M. (1988). *Temporary Equilibrium: Selected Readings*. Boston Academic Press.

Hicks, J. R. (1939). *Value and Capital*. Oxford University Press.

Iskhakov, F., T. H. Jørgensen, J. Rust, and B. Schjerning (2017). The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks. *Quantitative Economics 8*(2), 317–365.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2019). Credit supply and the housing boom. *Journal of Political Economy 127*(3), 1317–1350.

Kaas, L., G. Kocharkov, E. Preugschat, and N. Siassi (2021). Low homeownership in Germany – a quantitative exploration. *Journal of the European Economic Association 19*(1), 128–164.

Kaplan, G., K. Mitman, and G. L. Violante (2020). The housing boom and bust: Model meets evidence. *Journal of Political Economy 128*(9), 3285–3345.

Kiesl-Reiter, S., M. Lührmann, J. Shaw, and J. Winter (2024). The formation of subjective house price expectations. *Mimeo*.

Kindermann, F., J. Le Blanc, M. Piazzesi, and M. Schneider (2024). Learning about housing cost: Survey evidence from the German house price boom. *Mimeo*.

Kuchler, T., M. Piazzesi, and J. Stroebel (2023). Housing market expectations. In R. Bachmann, G. Topa, and W. van der Klaauw (Eds.), *Handbook of Economic Expectations*, pp. 163–191. Elsevier.

Kuchler, T. and B. Zafar (2019). Personal experiences and expectations about aggregate outcomes. *The Journal of Finance 74*(5), 2491–2542.

Landvoigt, T. (2017). Housing demand during the boom: The role of expectations and credit constraints. *The Review of Financial Studies 30*(6), 1865–1902.

Li, Z., S. Van Nieuwerburgh, and W. Renxuan (2023). Understanding rationality and disagreement in house price expectations. *NBER Working Papers 31516*, 1–63.

Manski, C. F. (2004). Measuring expectations. *Econometrica 72*(5), 1329–1376.

Molavi, P. (2019). Macroeconomics with learning and misspecification: A general theory and applications. *Mimeo*.

Nijskens, R. and M. Lohuis (2019). The housing market in major Dutch cities. In R. Nijskens, M. Lohuis, P. Hilbers, and W. Heeringa (Eds.), *Hot Property: The Housing Market in Major Cities*, pp. 23–35. Springer International Publishing.

Piazzesi, M. and M. Schneider (2009). Momentum traders in the housing market: Survey evidence and a search model. *American Economic Review 99*(2), 406–411.

Piazzesi, M. and M. Schneider (2016). Housing and macroeconomics. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Volume 2B, pp. 1547–1640. Elsevier.

Quintana, J. (2023). Do heterogeneous house-price expectations translate into heterogeneous mpcs? *Mimeo*.

Roth, C., M. Wiederholt, and J. Wohlfart (2023). The effects of monetary policy: Theory with measured expectations. *Mimeo*.

Simsek, A. (2021). The macroeconomics of financial speculation. *Annual Review of Economics 13*, 335–369.

Storesletten, K., C. I. Telmer, and A. Yaron (2004). Consumption and risk sharing over the life cycle. *Journal of Monetary Economics 51*(3), 609–633.

Zhang, H., A. R. Conn, and K. Scheinberg (2010). A derivative-free algorithm for least-squares minimization. *SIAM Journal on Optimization 20*(6), 3555–3576.

Appendix

Data Description and Sources

All data is aggregated at the household level and defined in annual terms unless otherwise stated. The main sample is from 2004 to 2017; The year 2004 is the first year in which questions on households' expectations of house prices were included in the sample. Households with negative net worth are dropped from the sample. The sample is further selected by dropping the bottom and top one percent of all expectations questions in order to eliminate extreme values. The household responses in the year in which the current accommodation was purchased and the year in which it was moved into underwent an error-correction phase to ensure they are weakly increasing and complete for the period of household participation; when the correct response is not obvious the observation has been dropped. Regarding the temporary equilibrium simulations, only households for which there is data on all state variables and who participated in at least two surveys are included in the analysis.

- Short-term Market House Price Expectations: Expected average change in house prices in the next two years; in annual percent. Source: DNB Household Survey (WOD205,WOD206, WOD44P,WOD44Q).
- Long-term House Price Expectations: Expected average increase in house prices over a period of ten years; in annual percent. Source: DNB Household Survey (WOD207,WOD44RA).
- Net income: Total net income minus income from interest and real estate income; specified in thousands of 2002 euros. Source: DNB Household Survey.
- Assets: Total assets excluding primary owned house; specified in thousands of 2002 euros. Source: DNB Household Survey.
- Housing: Value of primary owned house; specified in thousands of 2002 euros. Source: DNB Household Survey.
- Mortgage: Total value outstanding mortgages; specified in thousands of 2002 euros. Source: DNB Household Survey.
- Rent: Total rental expenditure; specified in thousands of 2002 euros. Source: DNB Household Survey (WOD205,WOD206).
- Age: Age of the household head. Source: DNB Household Survey.
- Household size: Number of household members. Source: DNB Household Survey.

- College: Dummy variable indicating if head of household has attended college. Source: DNB Household Survey.
- Retired: Dummy variable indicating if head of household is retired. Source: DNB Household Survey.
- Rural: Dummy variable indicating if household is in a rural region. Source: DNB Household Survey.
- Province: Variable denoting the province where the household is located. Source: DNB Household Survey.
- Home Adjustment Indicator: Dummy variable indicating if the household moved in the period; it is constructed using the variables indicating when the current accommodation was purchased or moved into. Source: DNB Household Survey.
- House Prices: Price index for housing in the Netherlands. Source: ECB's *Statistical Data Warehouse*.

Supplementary Appendix
—For Online Publication—

Analytical Derivations in Two-Period Model

The Problem

Household i lives for two periods and has preferences

$$u(c_0, c_1) = u(c_0) + \beta u(c_1) = \ln(c_0) + \beta \ln(c_1),$$

where β is the discount factor and c consumption. The household is endowed with some initial assets $a_0 \geq 0$ and earns a fixed exogenous income of y in both periods. The household can invest either in a financial asset a at price $q \equiv \frac{1}{R}$ or housing h at price p_0 and has some initial endowment of the liquid asset a_0. The period budget constraints are

$$c_0 + qa_1 + p_0 h_1 = y + a_0, \qquad c_1 \leq a_1 + p_1^i h_1 + y$$

We assume that the household can not short housing, i.e. we have $h_1 \geq 0$. Moreover, we also have a debt to income (DTI) constraint

$$-a_1 \leq \gamma y = \Gamma.$$

Thus, in total there are 4 constraints: 2 period budget constraints, which we know will be binding and the following non-negativity and debt to income constraints:

1. $a_1 \geq -\gamma y$
2. $p_0 h_1 \geq 0$

First-Order and Complementary Slackness Conditions

Attaching λ_i, $i = 0, 1, 2, 3$ to the four constraints gives the Lagrangian

$$\mathcal{L} = \max_{c_0, c_1, a_1, h_1} u(c_0) + \beta u(c_1) + \lambda_0[y + a_0 - c_0 - qa_1 - p_0 h_1] \\ + \lambda_1[y + a_1 + p_1^i h_1 - c_1] + \lambda_2[a_1 + \gamma y] + \lambda_3 p_0 h_1, \quad (11)$$

which gives the first-order conditions

$$\begin{aligned}
c_0 &: \quad u'(c_0) = \lambda_0 && \text{(12a)} \\
c_1 &: \quad \beta u'(c_1) = \lambda_1 && \text{(12b)} \\
a_1 &: \quad -\lambda_0 q + \lambda_1 + \lambda_2 = 0 && \text{(12c)} \\
h_1 &: \quad -\lambda_0 p_t + \lambda_1 p_1^i + \lambda_3 p_0 = 0. && \text{(12d)}
\end{aligned}$$

Rearranging (12c) and (12d) and dividing the latter by p_0 gives

$$\begin{aligned}
-\lambda_0 q + \lambda_1 &= -\lambda_2 && \text{(13a)} \\
-\lambda_0 + \lambda_1 \frac{p_1^i}{p_t} &= -\lambda_3. && \text{(13b)}
\end{aligned}$$

By no satiation and the lower Inada condition on marginal utility we know that budget constraints will bind and therefore λ_0 and λ_1 will be positive. The remaining complementary slackness conditions are:

$$\begin{aligned}
\lambda_2 &: \quad \lambda_2[a_1 + \gamma y] = 0 && \text{(14a)} \\
\lambda_3 &: \quad \lambda_3 p_0 h_1 = 0 && \text{(14b)}
\end{aligned}$$

Solution

We have 4 cases to consider. Foreshadowing a bit the results, we will sort them according to first whether or not saving in the financial asset happen in equilibrium or not. This is essentially a condition relating the interest rate to endowments and the discount factor. The focus of our paper is on the effect of house price expectations, therefore we sort the cases then in increasing order of house price expectations. Since housing in this simplified model is essentially just a second asset, households demand no (a positive amount of) housing if they expect its return to be below (above) that of the financial asset.

Case 1: a_1 interior and $h_1 = 0$. Here $\lambda_2 = 0$ but $\lambda_3 > 0$ and $a_1 > -\gamma y$, $h_1 = 0$. Rewrite (13a)

$$\lambda_0 = \frac{\lambda_1}{q} \tag{15}$$

and use it in (13b) to get

$$\lambda_1 \left(-R + \Delta P_1^i\right) = -\lambda_3.$$

Since the RHS is negative, it must be the case that

$$R > \Delta P_1^i. \tag{16}$$

Thus, if the return on financial assets exceeds the (expected) return on housing, the household invests only in the financial asset. Equation (13a) also implies an Euler equation in terms of financial assets.

$$\begin{aligned} u'(c_0) &= \beta R u'(c_1) \\ c_1 &= \beta R c_0, \end{aligned} \tag{17}$$

where the latter follows from log utility. Using this in the present value budget constraint

$$c_0 + qc_1 = a_0 + (1+q)y$$

we obtain, after some transformations, the equilibrium consumption decisions

$$c_0 = \frac{1}{1+\beta}\left(a_0 + y\left(1+q\right)\right) \tag{18a}$$

$$c_1 = \frac{\beta}{1+\beta}R\left(a_0 + y\right) + \frac{\beta}{1+\beta}y. \tag{18b}$$

This solution requires $a_1 > -\gamma y$. Inserting the solution for c_0 into the period budget constraint (recall $h_1 = 0$) yields

$$\begin{aligned} a_1 &= R\left[y + a_0 - c_0\right] = R\left[y + a_0 - \frac{1}{1+\beta}\left(a_0 + y\left(1+q\right)\right)\right] \\ &= \frac{R}{1+\beta}\left[\beta(y + a_0) - qy\right] \end{aligned}$$

from which we get

$$\begin{aligned} a_1 \geq -\gamma y \quad \Leftrightarrow \quad \frac{R}{1+\beta}\left[\beta(y+a_0) - qy\right] &\geq -\gamma y \\ \Leftrightarrow \quad \beta a_0 &\geq [-q(1+\beta)\gamma + q - \beta]y \\ \Leftrightarrow \quad a_0 &\geq \frac{1}{\beta}\left[-q(1+\beta)\gamma + q - \beta\right]y \end{aligned} \tag{19}$$

If $R\beta = 1$ and $\gamma = 0$, i.e. no borrowing is allowed, this boils down to $a_0 \geq 0$ which is intuitive since $R\beta = 1$ implies perfect consumption smoothing and period incomes are identical. Savings in period 1 are only positive if initial wealth is positive. If (19) does not hold, savings will be at the lower bound $a_1 = -\gamma y$, which is analyzed in case 3 below. Condition (19) can also be written as a requirement on the interest rate

$$R \geq \frac{1-(1+\beta)\gamma}{\beta} \frac{y}{y+a_0}$$

Case 2: both interior. Suppose $\lambda_2 = \lambda_3 = 0$, i.e. $a_1 > -\gamma y$ and $h_1 > 0$, then (13a) and (13b) imply

$$\begin{aligned} -\lambda_0 q + \lambda_1 &= 0 & \text{(20a)} \\ -\lambda_0 + \lambda_1 \frac{p_1^i}{p_t} &= 0 & \text{(20b)} \end{aligned}$$

and thus

$$\begin{aligned} &\Rightarrow \quad \frac{1}{q} = \frac{p_1^i}{p_0} \\ &\Leftrightarrow \quad R = \Delta P_1^i. \end{aligned}$$

Thus, an interior solution, where the household invests in both can only occur if the rate of returns are equal. In this case, consumptions are determined but portfolio choice is (within the bounds of the constraints) indeterminate. The Euler equation is standard

$$u'(c_0) = \beta R u'(c_1) = \beta \Delta P_1^i u'(c_1) \tag{21}$$

Inserting this into the intertemporal budget constraint yields the same allocations for consumption as (18a) and (18b). Housing $h_1 > 0$ and financial assets $a_1 > -\gamma y$ on the other hand are indeterminate, only the sum of the two $a_1 + p_0 h_1$ is determined. Equation (19) is still the relevant requirement for the lower bound on initial wealth for this equilibrium to occur. For simplicity, we assume that the household invests only in the financial asset when the returns are equal so that housing demand is

$$h_1\left(R, \Delta P_1^i\right) = 0 \Leftrightarrow \Delta P_1^i \leq R \tag{22}$$

Case 3: both at lower constraint $h_1 = 0$ and $a_1 = -\gamma y$. Suppose $\lambda_2 > 0$ and $\lambda_3 > 0$, i.e. $h_1 = 0$ and $a_1 = -\gamma y$, then (13a) and (13b) become

$$\begin{aligned} \lambda_0 q &= \lambda_1 + \lambda_2 & \text{(23a)} \\ \lambda_0 &= \lambda_1 \frac{p_1^i}{p_t} + \lambda_3. & \text{(23b)} \end{aligned}$$

Inserting marginal utilities, we get as Euler equations

$$u'(c_0) = \frac{1}{q}\beta u'(c_1) + \frac{\lambda_2}{q} \tag{24a}$$

$$u'(c_0) = \frac{p_1^i}{p_t}\beta u'(c_1) + \lambda_3. \tag{24b}$$

Since the last terms in both rows are positive and $c_0 = y + a_0 + \gamma y$ and $c_1 = y - R\gamma y$ implies that $u'(c_0) < u'(c_1)$, it must be that

$$\beta < \max\left[R, \Delta P_1^i\right] \tag{25}$$

From case 1, we know that in addition the initial wealth can not be too large, to be precise

$$a_0 < \frac{1}{\beta}[-q(1+\beta)\gamma + q - \beta]y \tag{26}$$

Since the returns and initial wealth are low, the household would like to borrow at the going rates in either the financial asset or housing but cannot.

Case 4: h_1 interior but $a_1 = -\gamma y$. Suppose $\lambda_2 > 0$ but $\lambda_3 = 0$, i.e. $a_1 = -\gamma y$ and $0 < h_1$, then (13b) implies

$$\lambda_0 = \lambda_1 \Delta P_1^i. \tag{27}$$

Combining the above with the FOCs for consumption, we get the Euler equation

$$u'(c_0) = \beta \Delta P_1^i u'(c_1) \tag{28}$$

with the return on housing as interest rate factor. This happens because (13a) and (13b) imply

$$\lambda_0 q = \lambda_1 + \lambda_2 \tag{29a}$$

$$\lambda_0 \frac{p_t}{p_1^i} = \lambda_1. \tag{29b}$$

Since $\lambda_2 > 0$ it must be that

$$\begin{aligned} q &< \frac{p_t}{p_1^i} \\ &\Rightarrow \Delta P_1^i > R, \end{aligned}$$

so that the return on housing exceeds the return on financial assets.

Since borrowing is at its maximum, let's first derive c_0, c_1 as a function of h_1 from the resource constraints:

$$c_0 = y + a_0 + q\gamma y - p_0 h_1 \tag{30a}$$

$$c_1 = y - \gamma y + p_1^i h_1. \tag{30b}$$

Next, use the above in the Euler equation to get

$$\begin{aligned}
c_1 &= \beta \Delta P_1^i c_0 \\
\Leftrightarrow \quad y - \gamma y + p_1^i h_1 &= \beta \Delta P_1^i (y + a_0 + q\gamma y - p_0 h_1) \\
\Leftrightarrow \quad p_1^i h_1 + \beta \Delta P_1^i p_0 h_1 &= \beta \Delta P_1^i (y + a_0 + q\gamma y) - y + \gamma y \\
\Leftrightarrow \quad p_1^i h_1 &= \frac{1}{1+\beta}[\beta \Delta P_1^i (y + a_0 + q\gamma y) + (\gamma - 1) y] \\
\Leftrightarrow \quad h_1 &= \frac{1}{1+\beta}\left[\frac{1}{p_0}\beta(y + a_0 + q\gamma y) + \frac{1}{p_1^i}(\gamma - 1) y\right]
\end{aligned}$$

and thus

$$h_1 = \frac{1}{p_0}\frac{1}{1+\beta}\left[\beta(y + a_0 + q\gamma y) - \frac{1}{\Delta P_1^i}(1-\gamma) y\right]. \tag{31}$$

This implies that

$$c_0 = y + a_0 + \gamma y - p_0 h_1 = y + a_0 + \gamma y - \frac{1}{1+\beta}\left[\beta(y + a_0 + q\gamma y) - \frac{1}{\Delta P_1^i}(1-\gamma) y\right]. \tag{32}$$

Now housing has to be positive, which requires the term in brackets in (31) to be positive

$$\beta(y + a_0 + q\gamma y) - \frac{1}{\Delta P_1^i}(1-\gamma) y \geq 0 \tag{33}$$

which implies

$$\begin{aligned}
\beta(y + a_0 + q\gamma y) &\geq \frac{1}{\Delta P_1^i}(1-\gamma) y \\
\Leftrightarrow \quad a_0 &\geq \frac{(1-\gamma)}{\beta \Delta P_1^i} y - (1 + q\gamma) y \\
\Leftrightarrow \quad a_0 &\geq \left[\frac{(1-\gamma)}{\beta \Delta P_1^i} - (1 + q\gamma)\right] y \\
\Leftrightarrow \quad a_0 &\geq \left[\frac{(1-\gamma) - (1 + q\gamma)\beta \Delta P_1^i}{\beta \Delta P_1^i}\right] y
\end{aligned}$$

Equation (31) is also concave in house price expectations for $\gamma \in (0, 1)$, since (ignoring the constant in front)

$$\frac{\partial h}{\partial \Delta P_1^i} = -(-1)\frac{(1-\gamma)y^2}{[\Delta P_1^i]} > 0$$

and

$$\frac{\partial^2 h}{\partial (\Delta P_1^i)^2} = -2\frac{(1-\gamma)y^3}{[\Delta P_1^i]} < 0.$$